Secure Recursive State Estimation for Singularly Perturbed Discrete Sequential Systems Under Round-Robin-Like Multichannel Access Policy

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***Abstract*— This paper investigates the secure state estimation problem for time-varying singularly perturbed discrete sequential systems (SPDSSs) under bandwidth-constrained and multichannel-enabled wireless communication environment. Firstly, to avoid the potential data collision as well as fully utilize the available wireless channels, a round-robin-like multichannel access policy (RRL-MAP) is presented for scheduling the transmission of measurement signals. The considered SPDSS is further assumed to be compromised by deception attack, and then the measurement outputs are updated based on the RRL-MAP and the attack driven by a Bernoulli process. In light of the constructed measurement model, distributed recursive state estimators are designed with the proposing of an algorithm that determines the estimators' gains. Finally, the efficiency of the developed state estimation method for the envisioned SPDSS is verified by both theoretical analysis and simulation experiments.**

***Note to Practitioners*—State estimation for SPDSSs is critical given the widespread application of the systems, however it is challenged by limited communication resource and stochastic cyber attacks. Traditional RR protocol is viewed as a desirable method for mitigating the limitation of network bandwidth, but which becomes inefficient under practical multichannel communication scenario. In addition, deception attack is acknowledged as a kind of hardly detected cyber attacks, and will seriously threaten the data integrity. In view of this, this paper presents the RRL-MAP to guarantee efficient data transmission by appropriately scheduling the available communication channels, and uses Bernoulli process to properly describe deception attack. Then, by further considering the structural characteristics of SPDSSs, distributed recursive state estimators are designed to achieve desired system performance. The effectiveness of the estimators is also comprehensively evaluated, which validates the practical applicability of the proposed state estimation method.**

***Index Terms*— Singularly perturbed discrete sequential systems (SPDSSs), recursive state estimation, round-robin-like multichannel access policy (RRL-MAP), deception attack.**

I. Introduction

DISCRETE sequential systems (DSSs) are generally composed by a group of subsystems which are orderly connected based on certain topologies depicted by directed graphs. Given the structural characteristic of DSSs, many practical systems, such as multi-state series systems and biological systems, can be effectively modeled by DSSs, and thus great attentions have been paid on DSSs [1], [2]. In engineering applications, it is important to obtain accurate states of DSSs which represent the dynamic information of the systems. However, the exact state acquiring can hardly be achieved due to various internal and external limitations, which raises the problem of state estimation based on available measurements [3], [4], [5]. In the existed literature, lots of strategies on state estimation have been reported. For example, H_∞ state estimation issue was addressed for coupled stochastic complex networks under periodical communication protocol in [6]; considering the time-varying characteristic of network topology, a quantization-based recursive state estimation scheme was proposed for discrete stochastic complex networks with uncertain inner coupling and error-variance constraints in [7]; focusing on time-varying DSSs, distributed Kalman filter and recursive filter under different constraints were developed in [1] and [2], respectively.

Despite the significant contributions of the aforementioned works to the state estimation problem, but all of them are based on one-time-scale systems, and thus not suitable for many practical systems presenting two-time-scale feature. Two-time-scale systems mean that the systems are governed by both fast and slow dynamics, and customarily are named as singularly perturbed systems (SPSs) [8]. For formally depicting SPSs, a singular perturbation parameter (SPP) with small positive value is generally introduced to differentiate the two kinds of dynamics [9], [10]. Up to now, many researches on state estimation over SPSs have been conducted. For instance, the authors in [11] proposed an integrated approach to handle both the exponential synchronization and state estimation

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problems for nonlinear singularly perturbed complex networks; an asynchronous proportional-integral observer-based state estimation scheme was designed for singularly perturbed complex networks subject to cyber attacks in [12]; state estimation problem for a class of discrete-time SPSs with distributed time-delays was investigated in [13]. Nevertheless, to the best of our knowledge, state estimation over singularly perturbed DSSs, referred to as SPDSSs, has not been explored yet. Actually, given the two-time-scale system dynamics and orderly functional relationship among subsystems, additional difficulties will be encountered in state estimation for SPDSSs, which is the primary motivation of this work.

Besides the above mentioned structural challenges, the potential data collision incurred by limited communication resource also can not be ignored while conducting state estimation. As we known, multiple sensors are now commonly used to acquire multidimensional information of a specific subsystem [14], and then data scheduling policy is needed to realize conflict-free network access under bandwidth-constrained wireless communication environment [15]. The typical data scheduling protocols include try-once-discard (TOD) protocol [16], [17], stochastic communication (SC) protocol [8], [18] and round-robin (RR) protocol [19]. From the perspectives of fairness and cost-efficiency, RR protocol is more favorable, and thus many RR-based state estimation methods have been proposed by scholars. To mention a few, the RR-based state estimation problem was studied over nonlinear dynamical networks with time-varying delays and disturbance in [20]; taking the effect of SPP into account, a H_∞ state estimation method for discrete-time complex networks with RR protocol was proposed in [21]; the authors in [22] designed a recursive state estimator for multirate and multisensor systems with time-delays based on RR protocol. We notice that all the discussed works assume that only one communication channel is available at each time instant and then RR protocol is employed to avoid potential data collision. However, for IEEE 802.11 based wireless sensor networks (WSNs), multiple nonoverlapping channels are generally enabled [23], [24], under which traditional RR protocol becomes inefficient in channel utilization. Therefore, this paper will present a RR-like multichannel access policy (referred to as RRL-MAP) so as to effectively support state estimation for SPDSSs under multichannel communication scenario.

Following the RRL-MAP, the measurement signals generated from sensors can be released into multiple nonoverlapping wireless channels without data collision, but the signals may not be safely received for state estimation due to the prevalence of cyber attacks [25]. It is acknowledged that the common types of cyber attacks are denial of service (DoS) attack, replay attack and deception attack [26], [27]. Comparing with the first two types, deception attack is more difficult to be detected since that destination terminals can always receive the real-time updating signals but actually the original data has been maliciously modified. As such, it is critical to take deception attack into consideration while studying state estimation issue, and some efforts have been put into practice. For example, a multipliers-based distributed state estimation method for smart grid subject to deception attack was developed in [28]; considering the influence of both deception attack and DoS attack, an event-based recursive state estimation scheme over stochastic complex dynamical networks was presented in [29]. But so far few works explore the deception attack tolerant state estimation problem over SPDSSs, not to mention under the envisioned RRL-MAP.

In this paper, under the scenario that the limited wireless bandwidth is divided into multiple nonoverlapping channels and the transmission of measurement signals is affected by deception attack, we will design distributed recursive state estimators for SPDSSs to assure systems' performance. The major contributions of the study can be highlighted as follows.

- For supporting state estimation over SPDSSs under bandwidth-limited and multichannel-enabled wireless communication environment, the RRL-MAP is presented to realize non-collision and efficient transmission of measurement signals via appropriately scheduling the nonoverlapping wireless channels.
- Based on the RRL-MAP and taking stochastic deception attack into consideration, recursive state estimators are designed after establishing a new measurement model.
- Under the developed estimators, an upper-bound for each estimation error covariance is derived, then an algorithm for determining the estimators' gains is proposed, which is followed by the effectiveness analysis of the algorithm.

The rest of the paper is organized as follows. In Section II, based on the introduction of the considered SPDSS, the envisioned RRL-MAP and deception attack, the studied problem is formulated as designing distributed recursive state estimators. The algorithm for deriving appropriate estimators' parameters is designed and evaluated in Section III. A simulation example is conducted in Section IV to further illustrate the performance of the presented state estimation method. The conclusion of the paper is given in Section V.

II. Problem Statement

In this section, we firstly demonstrate the structure characteristic of DSSs, and subsequently introduce the model of the considered SPDSS. Then, the envisioned RRL-MAP and deception attack are formally described, and the measurement signals are updated accordingly. Finally, distributed recursive estimators with desired performance are designed based on the system model and updated measurements.

A. System Model

As we mentioned before, the subsystems in DSSs are orderly connected and such relationships can be depicted by directed graphs. To specifically express the structure feature of DSSs, an example of a DSS consisting of 9 subsystems is presented in Fig. 1, where the i-th subsystem is denoted by S_i $(1 \leq i \leq 9)$. In the figure, the arc $< S_i, S_j >$ means that the local state of S_j is directly affected by that of S_i, it can thus be found that the state of each S_j can only be influenced by that of S_i $(i \leq j)$ and can not be affected by that of S_l $(l > j)$, which is the sequential characteristic of DSSs.

On the basis of the above illustration and introducing the concept of SPP, a SPDSS composed by N subsystems is

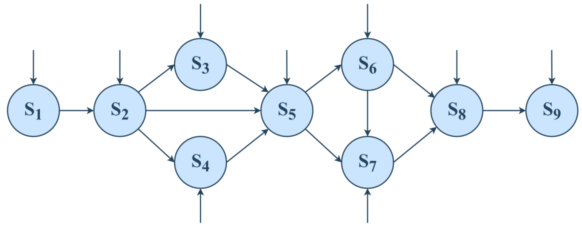

Fig. 1. An example of the structure of DSSs.

considered in this paper. Each subsystem S_i $(1 \leq i \leq N)$ is formulated as follows:

$$\begin{cases} \mathbf{x}_i(k+1) = A_i(k)H_\epsilon \mathbf{x}_i(k) + E_i(k)\omega_i(k) \\ \qquad\qquad +\Sigma_{j\in K_i} B_{i,j}(k)H_\epsilon \mathbf{x}_j(k), \\ \tilde{\mathbf{y}}_i(k) = C_i(k)H_\epsilon \mathbf{x}_i(k) + v_i(k), \end{cases} \tag{1}$$

where $\mathbf{x}_i(k) = col\{\mathbf{x}_{s,i}(k), \mathbf{x}_{f,i}(k)\} \in \mathbb{R}^{n_x}$ is the state vector of S_i, furthermore, $\mathbf{x}_{s,i}(k) \in \mathbb{R}^{n_s}$ and $\mathbf{x}_{f,i}(k) \in \mathbb{R}^{n_f}$ $(n_s + n_f = n_x)$ are the system slow and fast state vectors, respectively; $H_\epsilon = diag\{I_{n_s}, \epsilon I_{n_f}\}$, and the constant ϵ $(0 < \epsilon \ll 1)$ is the SPP; K_i denotes the set of subscripts of the other subsystems whose states directly affect the state of S_i; $\tilde{\mathbf{y}}_i(k) \in \mathbb{R}^{n_y}$ is the measurement output of S_i, and $A_i(k)$, $B_{i,j}(k)$, $C_i(k)$, $E_i(k)$ are known time-varying matrices with appropriate dimensions; the process noise $\omega_i(k)$ and the measurement noise $v_i(k)$ are assumed to be zero-mean Gaussian white noises with the following properties:

$$\mathbb{E}\{\omega_i(k)\} = \mathbb{E}\{v_i(k)\} = \mathbb{E}\{\omega_i(k)v_i^T(k)\} = 0,$$
$$\mathbb{E}\{\omega_i(k)\omega_i^T(k)\} = \mathcal{W}_i, \quad \mathbb{E}\{v_i(k)v_i^T(k)\} = \mathcal{V}_i,$$

where $\mathcal{W}_i$ and $\mathcal{V}_i$ are positive numbers. Based on the above illustration, it can be found that the major physical meanings of the SPDSS can be well demonstrated by Eq. (1). On one hand, the sequential characteristic of the system can be depicted by K_i, i.e., $j \in K_i$ only if S_j is in front of S_i and the local state of S_i is directly affected by that of S_j; on the other hand, the two-time-scale feature of the system is reflected by H_ϵ which is defined based on the SPP ϵ.

Considering that a distributed state estimation method will be proposed in this paper, i.e., a dedicated state estimator will be designed for each subsystem, without loss of generality, the following description and analysis will be focused on S_i.

B. The Envisioned RRL-MAP and Deception Attack

For S_i, it is supposed that m nonoverlapping channels are available in the WSN and n_y sensors are used for data sensing, so it has $\tilde{\mathbf{y}}_i(k) = col\{\tilde{\mathbf{y}}_{i,1}(k), \tilde{\mathbf{y}}_{i,2}(k), \ldots, \tilde{\mathbf{y}}_{i,n_y}(k)\}$, where $\tilde{\mathbf{y}}_{i,r}(k)$ represents the r-th component of $\tilde{\mathbf{y}}_i(k)$ $(1 \leq r \leq n_y)$. Given the limitation of wireless bandwidth, we further assume that $m < n_y$ and then employ the RRL-MAP to arrange the specific data transmission. Let $\varepsilon(k)$ be the set of indexes of the sensors that are allowed to access the multichannel-enabled WSN at the time instant k $(k = 1, 2, \ldots)$, then the basis idea of the RRL-MAP can be depicted by the following equation.

$$\varepsilon(k) = \Big\{mod\ (m * (k-1) + s, n_y) + 1 | s = 0, 1, \ldots, m-1\Big\}, \tag{2}$$

where $mod\ (a, b)$ represents the remainder of a divided by b. According to Eq. (2), we have $\varepsilon(k) \subseteq \{1, 2, ..., n_y\}$ and $|\varepsilon(k)| = m$, moreover, it can be found that $\varepsilon(k)$ is a periodic sequence with the cycle of U time instants, where $U = lcm(m, n_y)/m$ and $lcm(a, b)$ denotes the least common multiple of a and b.

Remark 1: The RRL-MAP is based on traditional RR protocol, so it is essentially a periodic data scheduling policy but works for multichannel communication scenario. Under the RRL-MAP, all sensors will obtain equal opportunities for data transmission within each U time instants, and the wireless channels can be fully utilized without data collision by assigning each authorized sensor with a dedicated channel at each time instant.

Following the RRL-MAP, the updating on $\tilde{\mathbf{y}}_{i,r}(k)$ can be described as:

$$\bar{\mathbf{y}}_{i,r}(k) = \begin{cases} \tilde{\mathbf{y}}_{i,r}(k), & r \in \varepsilon(k), \\ 0, & r \notin \varepsilon(k). \end{cases} \tag{3}$$

We further define the following matrix:

$$\Phi_{o(k)} = \sum_{r\in\varepsilon(k)} \phi_r, \tag{4}$$

where $\phi_r = diag\{\delta(r-1)I, \delta(r-2)I, \ldots, \delta(r-n_y)I\}$, $\delta(\cdot) \in \{0, 1\}$ is a Kronecker delta function, and $o(k) = mod\ ((k-1), U) + 1$, then it can be gotten that:

$$\bar{\mathbf{y}}_i(k) = \Phi_{o(k)}\tilde{\mathbf{y}}_i(k), \tag{5}$$

where $\bar{\mathbf{y}}_i(k) = col\{\bar{\mathbf{y}}_{i,1}(k), \bar{\mathbf{y}}_{i,2}(k), \ldots, \bar{\mathbf{y}}_{i,n_y}(k)\}$.

Remark 2: In this study, zero-input strategy is adopted for measurements updating of sensors that are not permitted to access the WSN as shown in Eq. (3). Comparing with zero-order holder (ZOH) method which uses the stored most recent signals for data updating, zero-input strategy is more cost-efficient and mathematically convenient [30]. It is also worthy noting that zero-input strategy will not necessarily lead to a worse system performance than ZOH method which has been validated in [31] and [32].

For the considered deception attack, a Bernoulli variable $\psi_i(k)$ with the following probability:

$$\mathbf{Pr}\{\psi_i(k) = 1\} = \check{\psi}_i \quad (\mathbf{Pr}\{\psi_i(k) = 0\} = 1 - \check{\psi}_i),$$

is adopted to depict the stochastic property of the attack, where $\check{\psi}_i$ is a given parameter, then the attack-influenced measurement signal can be represented as:

$$\mathbf{y}_i(k) = \psi_i(k)\bar{\mathbf{y}}_i(k) + (1 - \psi_i(k))\Phi_{o(k)}\xi_i(k), \tag{6}$$

in which $\xi_i(k)$ is the attack function satisfying $||\xi_i(k)||^2 \leq \xi_i^*$ [33], $\mathbf{y}_i(k)$ is the signal that really received by the estimator of S_i.

Remark 3: As revealed by Eq. (6), the deception attack is launched intermittently, i.e., when $\psi_i(k) = 0$, the original data will be falsified by the attacker, otherwise the measurement signal will normally be transmitted to the estimator, which is consistent with the actual situation that the attacker cannot always obtain system information successfully. Moreover, $\check{\psi}_i$ is an empirical value which can be determined by long-term monitoring and evaluation [2]. The specific form of the attack function $\xi_i(k)$ can be diverse as long as the energy upper-bound ξ_i^* is maintained, actually the upper-bound is set for enhancing the concealment of the attack, and it can be evaluated by the defender via statistical experiments [34].

C. Recursive State Estimator

Based on the above description on the RRL-MAP and deception attack, a time-varying recursive estimator for S_i is constructed within the following two steps.

(1) Prediction:

$$\begin{aligned}\hat{\mathbf{x}}_i(k|k-1) &= A_i(k-1)H_\epsilon\hat{\mathbf{x}}_i(k-1|k-1)\\ &\quad + \Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon\hat{\mathbf{x}}_j(k-1|k-1),\end{aligned} \tag{7}$$

where $\hat{\mathbf{x}}_i(k|k-1)$ represents the one-step prediction.

(2) Updating:

$$\begin{aligned}\hat{\mathbf{x}}_i(k|k) &= \hat{\mathbf{x}}_i(k|k-1) + L_i(k)\\ &\quad \times \Big(\mathbf{y}_i(k) - \Phi_{o(k)}C_i(k)H_\epsilon\hat{\mathbf{x}}_i(k|k-1)\Big),\end{aligned} \tag{8}$$

where $\hat{\mathbf{x}}_i(k|k)$ is the estimation for $\mathbf{x}_i(k)$, and $L_i(k)$ is the estimator gain matrix which will be designed shortly.

According to the above definition, the one-step prediction error can be represented as:

$$e_i(k|k-1) = \mathbf{x}_i(k) - \hat{\mathbf{x}}_i(k|k-1), \tag{9}$$

and the state estimation error can be computed as:

$$e_i(k|k) = \mathbf{x}_i(k) - \hat{\mathbf{x}}_i(k|k). \tag{10}$$

By substituting Eqs. (1) and (7) into Eq. (9), it can be obtained that:

$$\begin{aligned}e_i(k|k-1) &= A_i(k-1)H_\epsilon e_i(k-1|k-1)\\ &\quad + \Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon e_j(k-1|k-1)\\ &\quad + E_i(k-1)\omega_i(k-1).\end{aligned} \tag{11}$$

Similarly, by recurring to Eqs. (1) and (8), Eq. (10) can be rewritten as:

$$\begin{aligned}e_i(k|k) &= \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)e_i(k|k-1)\\ &\quad - L_i(k)\Big((\psi_i(k)-1)\Phi_{o(k)}C_i(k)H_\epsilon\mathbf{x}_i(k)\\ &\quad + \psi_i(k)\Phi_{o(k)}v_i(k) + (1-\psi_i(k))\Phi_{o(k)}\xi_i(k)\Big).\end{aligned} \tag{12}$$

Furthermore, the covariance matrices of $e_i(k|k-1)$ and $e_i(k|k)$ can be denoted as:

$$\mathcal{P}_i(k|k-1) = \mathbb{E}\{e_i(k|k-1)e_i^T(k|k-1)\}, \tag{13}$$

and

$$\mathcal{P}_i(k|k) = \mathbb{E}\{e_i(k|k)e_i^T(k|k)\}, \tag{14}$$

respectively.

III. Main Results

In this section, an upper-bound for the state estimation error covariance matrix (i.e., $\mathcal{P}_i(k|k)$) is derived firstly, we then show that an appropriate estimator gain matrix $L_i(k)$ can be obtained by minimizing the trace of the presented upper-bound, the performance of the estimator with the designed parameter $L_i(k)$ is finally evaluated by theoretical analysis.

A. Estimator Design

Before proposing the specific algorithm for designing appropriate state estimator, i.e., deriving the estimator gain, the following lemmas are firstly listed.

Lemma 1: [35] For $\alpha > 0$, matrices $\mathcal{B}$ and $\mathcal{C}$ with appropriate dimensions, it has:

$$\mathcal{B}\mathcal{C}^T + \mathcal{C}\mathcal{B}^T \leq \alpha\mathcal{C}\mathcal{C}^T + \alpha^{-1}\mathcal{B}\mathcal{B}^T. \tag{15}$$

Lemma 2: [29] For any matrices $\mathcal{O}$, $\mathcal{M}$, $\mathcal{Z}$, and $\mathcal{Y}$ with compatible dimensions, the following equations hold.

$$\begin{aligned}\frac{\partial \mathbf{tr}\Big((\mathcal{O}-\mathcal{Z}\mathcal{M})\mathcal{Y}(\mathcal{O}-\mathcal{Z}\mathcal{M})^T\Big)}{\partial\mathcal{Z}} &= -2(\mathcal{O}\mathcal{M}^T + \mathcal{Z}\mathcal{M}\mathcal{M}^T),\\ \frac{\partial \mathbf{tr}\Big((\mathcal{O}\mathcal{Z}\mathcal{M})\mathcal{Y}(\mathcal{O}\mathcal{Z}\mathcal{M})^T\Big)}{\partial\mathcal{Z}} &= 2\mathcal{O}^T\mathcal{O}\mathcal{Z}\mathcal{M}\mathcal{Y}\mathcal{M}^T.\end{aligned} \tag{16}$$

Theorem 1: For given positive scalars ϵ, α_l (l = 1-4) and λ, if the following two coupled recursive matrix equations:

$$\begin{aligned}&\Im_i(k|k-1)\\ &\quad = (1+\lambda)A_i(k-1)H_\epsilon\Im_i(k-1|k-1)\\ &\qquad \times H_\epsilon^TA_i^T(k-1) + (1+\lambda^{-1})\Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon\\ &\qquad \times \Im_j(k-1|k-1)\Sigma_{j\in K_i}H_\epsilon^TB_{i,j}^T(k-1)\\ &\qquad + E_i(k-1)\mathcal{W}_iE_i^T(k-1),\end{aligned} \tag{17}$$

and

$$\begin{aligned}\Im_i(k|k) &= \Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\\ &\quad \times \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\Im_i(k|k-1)\\ &\quad \times \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T + (1+\alpha_1)\\ &\quad \times \Big(\check{\psi}_i + \alpha_3^{-1} + \alpha_4 - 1\Big)L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\\ &\quad \times \Im_i(k|k-1)H_\epsilon^TC_i^T(k)\Phi_{o(k)}^TL_i^T(k) + (1+\alpha_1^{-1})\\ &\quad \times \Big(\check{\psi}_i + \alpha_3^{-1} + \alpha_4 - 1\Big)L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\\ &\quad \times \mathbb{E}\{\hat{\mathbf{x}}_i(k|k-1)\hat{\mathbf{x}}_i^T(k|k-1)\}H_\epsilon^TC_i^T(k)\Phi_{o(k)}^T\\ &\quad \times L_i^T(k) + \Big(1+\alpha_2^{-1}+\alpha_4^{-1}\Big)\Big(1-\check{\psi}_i\Big)L_i(k)\Phi_{o(k)}\\ &\quad \times (\xi_i^*I)\Phi_{o(k)}^TL_i^T(k) + \check{\psi}_iL_i(k)\Phi_{o(k)}\mathcal{V}_i\Phi_{o(k)}^TL_i^T(k),\end{aligned} \tag{18}$$

with initial condition:

$$0 \leq \mathcal{P}_i(0|0) \leq \Im_i(0|0), \tag{19}$$

have positive-definite solutions $\Im_i(k|k-1)$ and $\Im_i(k|k)$, respectively, then $\Im_i(k|k)$ is an upper-bound of $\mathcal{P}_i(k|k)$, i.e., $\mathcal{P}_i(k|k) \leq \Im_i(k|k)$.

Proof: Firstly, according to Eqs. (11)-(14), the one-step prediction error covariance matrix $\mathcal{P}_i(k|k-1)$ and estimation error covariance matrix $\mathcal{P}_i(k|k)$ can be further expressed as:

$$\begin{aligned}
&\mathcal{P}_i(k|k-1)\\
&\quad=\mathbb{E}\{e_i(k|k-1)e_i^T(k|k-1)\}\\
&\quad=A_i(k-1)H_\epsilon\mathcal{P}_i(k-1|k-1)H_\epsilon^T A_i^T(k-1)\\
&\qquad+\Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon\mathcal{P}_j(k-1|k-1)\\
&\qquad\times\Sigma_{j\in K_i}H_\epsilon^T B_{i,j}^T(k-1)+\Sigma_{j\in K_i}A_i(k-1)\\
&\qquad\times H_\epsilon\mathcal{P}_{i,j}(k-1|k-1)H_\epsilon^T B_{i,j}^T(k-1)\\
&\qquad+\Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon\mathcal{P}_{j,i}(k-1|k-1)\\
&\qquad\times H_\epsilon^T A_i^T(k-1)+E_i(k-1)\mathcal{W}_i E_i^T(k-1),
\end{aligned}\tag{20}$$

$$\begin{aligned}
&\mathcal{P}_i(k|k)\\
&\quad=\mathbb{E}\{e_i(k|k)e_i^T(k|k)\}\\
&\quad=\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\mathcal{P}_i(k|k-1)\\
&\qquad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T+\Big(\check{\psi}_i-1\Big)L_i(k)\\
&\qquad\times\Phi_{o(k)}C_i(k)H_\epsilon\mathbb{E}\{\mathbf{x}_i(k)\mathbf{x}_i^T(k)\}H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T\\
&\qquad\times L_i^T(k)+\Big(1-\check{\psi}_i\Big)L_i(k)\Phi_{o(k)}\mathbb{E}\{\xi_i(k)\xi_i^T(k)\}\Phi_{o(k)}^T\\
&\qquad\times L_i^T(k)+\check{\psi}_i L_i(k)\Phi_{o(k)}\mathcal{V}_i\Phi_{o(k)}^T L_i^T(k)-\Big(1-\check{\psi}_i\Big)\\
&\qquad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\mathbb{E}\{e_i(k|k-1)\xi_i^T(k)\}\Phi_{o(k)}^T\\
&\qquad\times L_i^T(k)-\Big(1-\check{\psi}_i\Big)L_i(k)\Phi_{o(k)}\\
&\qquad\times\mathbb{E}\{\xi_i(k)e_i^T(k|k-1)\}\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\\
&\qquad-\Big(\check{\psi}_i-1\Big)L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\mathbb{E}\{\mathbf{x}_i(k)e_i^T(k|k-1)\}\\
&\qquad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T-\Big(\check{\psi}_i-1\Big)\\
&\qquad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\mathbb{E}\{e_i(k|k-1)\mathbf{x}_i^T(k)\}\\
&\qquad\times H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T L_i^T(k)+\Big(1-\check{\psi}_i\Big)L_i(k)\Phi_{o(k)}C_i(k)\\
&\qquad\times H_\epsilon\mathbb{E}\{\mathbf{x}_i(k)\xi_i^T(k)\}\Phi_{o(k)}^T L_i^T(k)+\Big(1-\check{\psi}_i\Big)L_i(k)\Phi_{o(k)}\\
&\qquad\times\mathbb{E}\{\xi_i(k)\mathbf{x}_i^T(k)\}H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T L_i^T(k),
\end{aligned}\tag{21}$$

where $\mathcal{P}_{i,j}(k-1|k-1)=\mathbb{E}\{e_i(k-1|k-1)e_j^T(k-1|k-1)\}$ and $\mathcal{P}_{j,i}(k-1|k-1)=\mathbb{E}\{e_j(k-1|k-1)e_i^T(k-1|k-1)\}$.

Then, the theorem is proved by using mathematical induction method. Specifically:

- In light of the initial condition given in Eq. (19), it is obvious that the proposition is valid when $k=0$.
- Assuming that $\mathcal{P}_i(k-1|k-1)\le\Im_i(k-1|k-1)$, we then prove that $\mathcal{P}_i(k|k)\le\Im_i(k|k)$. Firstly, based on Lemma 1, it has:

$$\begin{aligned}
&\Sigma_{j\in K_i}A_i(k-1)H_\epsilon\mathcal{P}_{i,j}(k-1|k-1)H_\epsilon^T B_{i,j}^T(k-1)\\
&\quad+\Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon\mathcal{P}_{j,i}(k-1|k-1)H_\epsilon^T A_i^T(k-1)\\
&\le\lambda A_i(k-1)H_\epsilon\mathcal{P}_i(k-1|k-1)H_\epsilon^T A_i^T(k-1)\\
&\quad+\lambda^{-1}\Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon\mathcal{P}_j(k-1|k-1)\\
&\quad\times H_\epsilon^T B_{i,j}^T(k-1).
\end{aligned}\tag{22}$$

Then, according to the assumption, the Eqs. (17) and (20), it can be concluded that $\mathcal{P}_i(k|k-1)\le\Im_i(k|k-1)$. By applying Lemma 1 and considering the upper-bound of the deception attack signal, i.e., $||\xi_i(k)||^2\le\xi_i^*$, which means that $\xi_i^T(k)\xi_i(k)\le\xi_i^T(k)\xi_i(k)I\le\xi_i^*I$, it can be obtained that:

$$\begin{aligned}
&\Big(1-\check{\psi}_i\Big)L_i(k)\Phi_{o(k)}\mathbb{E}\{\xi_i(k)\xi_i^T(k)\}\Phi_{o(k)}^T L_i^T(k)\\
&\qquad\le\Big(1-\check{\psi}_i\Big)L_i(k)\Phi_{o(k)}(\xi_i^*I)\Phi_{o(k)}^T L_i^T(k).
\end{aligned}\tag{23}$$

Furthermore, following the similar handling method, the inequalities below can be easily derived.

$$\begin{aligned}
&\mathbb{E}\{\mathbf{x}_i(k)\mathbf{x}_i^T(k)\}=(e_i(k|k-1)+\hat{\mathbf{x}}_i(k|k-1))\\
&\qquad\times(e_i(k|k-1)+\hat{\mathbf{x}}_i(k|k-1))^T\\
&\quad\le(1+\alpha_1)e_i(k|k-1)e_i^T(k|k-1)\\
&\qquad+(1+\alpha_1^{-1})\mathbb{E}\{\hat{\mathbf{x}}_i(k|k-1)\hat{\mathbf{x}}_i^T(k|k-1)\},
\end{aligned}\tag{24}$$

$$\begin{aligned}
&\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\mathbb{E}\{e_i(k|k-1)\xi_i^T(k)\}\\
&\qquad\times L_i^T(k)+L_i(k)\Phi_{o(k)}\mathbb{E}\{\xi_i(k)e_i^T(k|k-1)\}\\
&\qquad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\\
&\quad\le\alpha_2\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\mathcal{P}_i(k|k-1)\\
&\qquad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\\
&\qquad+\alpha_2^{-1}L_i(k)\Phi_{o(k)}(\xi_i^*I)\Phi_{o(k)}^T L_i^T(k),
\end{aligned}\tag{25}$$

$$\begin{aligned}
&L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\mathbb{E}\{\mathbf{x}_i(k)e_i^T(k|k-1)\}\\
&\qquad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\\
&\qquad+\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\mathbb{E}\{e_i(k|k-1)\mathbf{x}_i^T(k)\}\\
&\qquad\times H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T L_i^T(k)\\
&\quad\le\alpha_3\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\mathcal{P}_i(k|k-1)\\
&\qquad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\\
&\qquad+\alpha_3^{-1}L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\mathbb{E}\{\mathbf{x}_i(k)\mathbf{x}_i^T(k)\}\\
&\qquad\times H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T L_i^T(k),
\end{aligned}\tag{26}$$

$$\begin{aligned}
&L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\mathbb{E}\{\mathbf{x}_i(k)\xi_i^T(k)\}\Phi_{o(k)}^T L_i^T(k)\\
&\qquad+L_i(k)\Phi_{o(k)}\mathbb{E}\{\xi_i(k)\mathbf{x}_i^T(k)\}H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T L_i^T(k)\\
&\quad\le\alpha_4 L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\mathbb{E}\{\mathbf{x}_i(k)\mathbf{x}_i^T(k)\}H_\epsilon^T C_i^T(k)\\
&\qquad\times\Phi_{o(k)}^T L_i^T(k)+\alpha_4^{-1}L_i(k)\Phi_{o(k)}(\xi_i^*I)\Phi_{o(k)}^T L_i^T(k).
\end{aligned}\tag{27}$$

By summarizing Eqs. (18), (21), and (23)-(27), we can conclude that $\mathcal{P}_i(k|k)\le\Im_i(k|k)$.

So far, the theorem is proved. □

Remark 4: The upper-bound derived in Theorem 1 is dependent on some scalars, i.e., ϵ, λ and α_l ($l=$ 1-4). Generally, the SPP ϵ is non-adjustable for a given SPDSS, but the other parameters can be changed arbitrarily. Although it is hard to obtain close-form solutions of the optimal values of these adjustable parameters, the intelligent optimization algorithms, such as particle swarm algorithm and genetic algorithm, with the objective $min_{\lambda,\alpha_1,\alpha_2,\alpha_3,\alpha_4}\Im_i(k|k)$, can be adopted to minimize the upper-bound so as to improve state estimation performance.

Theorem 2: For given positive scalars ϵ, α_l (l = 1-4) and λ, the upper-bound $\Im_i(k|k)$ for the estimation error covariance can be minimized by designing the estimator parameter as:

$$L_i(k) = \Psi_i(k)\Pi_i^{-1}(k), \tag{28}$$

where

$$\begin{aligned}\Psi_i(k) = &\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\Im_i(k|k-1)\\ &\times H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T,\\ \Pi_i(k) = &\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\Phi_{o(k)}C_i(k)H_\epsilon\\ &\times \Im_i(k|k-1)H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T + (1+\alpha_1)\\ &\times \Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4-1\Big)\Phi_{o(k)}C_i(k)H_\epsilon\\ &\times \Im_i(k|k-1)H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T + (1+\alpha_1^{-1})\\ &\times \Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4-1\Big)\Phi_{o(k)}C_i(k)H_\epsilon\\ &\times \mathbb{E}\{\hat{\mathbf{x}}_i(k|k-1)\hat{\mathbf{x}}_i^T(k|k-1)\}H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T\\ &+\Big(1+\alpha_2^{-1}+\alpha_4^{-1}\Big)\Big(1-\check{\psi}_i\Big)\Phi_{o(k)}(\xi_i^* I)\Phi_{o(k)}^T\\ &+\check{\psi}_i\Phi_{o(k)}\mathcal{V}_i\Phi_{o(k)}^T.\end{aligned}$$

Proof: By recurring to Lemma 2, and taking partial derivative of the trace of $\Im_i(k|k)$ with respect to $L_i(k)$, one has:

$$\begin{aligned}&\frac{\partial \mathbf{tr}(\Im_i(k|k))}{\partial L_i(k)}\\ &= 2\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\\ &\quad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\Im_i(k|k-1)\\ &\quad\times H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T + 2(1+\alpha_1)\Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4-1\Big)\\ &\quad\times L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Im_i(k|k-1)H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T\\ &\quad+2(1+\alpha_1^{-1})\Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4-1\Big)L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\\ &\quad\times\mathbb{E}\{\hat{\mathbf{x}}_i(k|k-1)\hat{\mathbf{x}}_i^T(k|k-1)\}H_\epsilon^T C_i^T(k)\Phi_{o(k)}^T\\ &\quad+2\Big(1+\alpha_2^{-1}+\alpha_4^{-1}\Big)\Big(1-\check{\psi}_i\Big)L_i(k)\Phi_{o(k)}(\xi_i^* I)\Phi_{o(k)}^T\\ &\quad+2\check{\psi}_i L_i(k)\Phi_{o(k)}\mathcal{V}_i\Phi_{o(k)}^T.\end{aligned}\tag{29}$$

Letting $\frac{\partial \mathbf{tr}(\Im_i(k|k))}{\partial L_i(k)} = 0$, we can get $L_i(k)$ as presented in Eq. (28). Thus, the proof is completed. □

On the basis of Theorem 2 which shows that the estimator gain matrix $L_i(k)$ can be obtained by solving $\frac{\partial \mathbf{tr}(\Im_i(k|k))}{\partial L_i(k)} = 0$, the proposed algorithm for designing secure recursive state estimator for S_i with a time window T is specifically described by Algorithm 1.

B. Effectiveness Analysis

In this subsection, the effectiveness of the designed state estimation algorithm is validated by showing that the estimation error $e_i(k|k)$ derived under the algorithm is exponentially bounded in the mean square. For this, the lemma and assumption below are firstly listed.

Algorithm 1 The Proposed Estimator Design Algorithm

1 Initializing $\mathbf{x}_i(0)$, $\hat{\mathbf{x}}_i(0|0)$ and $\Im_i(0|0)$;
2 **for** $k = 1;\ k \le T;\ k = k+1$ **do**
3 $\quad\mathbf{x}_i(k) = A_i(k-1)H_\epsilon\mathbf{x}_i(k-1) + E_i(k-1)\omega_i(k-1)$ $+\Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon\mathbf{x}_j(k-1)$;
4 $\quad\tilde{\mathbf{y}}_i(k) = C_i(k)H_\epsilon\mathbf{x}_i(k) + \upsilon_i(k)$;
5 $\quad U = lcm(m, n_y)/m$; $o(k) = mod\ ((k-1), U) + 1$;
6 $\quad$Computing $\varepsilon(k)$ and $\Phi_{o(k)}$ based on Eq. (2) and Eq. (4), respectively;
7 $\quad$Calculating $\mathbf{y}_i(k)$ according to Eq. (6);
8 $\quad$Deriving $\hat{\mathbf{x}}_i(k|k-1)$ and $\Im_i(k|k-1)$ based on Eq. (7) and Eq. (17), respectively;
9 $\quad$Obtaining $L_i(k)$ according to Eq. (28);
10 $\quad$Calculating $\hat{\mathbf{x}}_i(k|k)$ and $\Im_i(k|k)$ based on Eq. (8) and Eq. (18), respectively.
11 **end**

Lemma 3: [36] If the following two inequalities:

$$j_*\|\psi_k\|^2 \le W(\psi_k) \le j^*\|\psi_k\|^2, \tag{30}$$

and

$$\mathbb{E}\{W(\psi_{k+1}|\psi_k)\} - W(\psi_k) \le \Upsilon - fW(\psi_k), \tag{31}$$

hold with a stochastic process $W(\psi_k)$ and scalars j^*, j_*,$\Upsilon >$ 0, $0 \le f \le 1$, then for any $k \ge 0$, it has:

$$\mathbb{E}\{\|\psi_k\|^2\} \le \frac{j^*}{j_*}\mathbb{E}\{\|\psi_0\|^2\}(1-\Upsilon)^k + \frac{f}{j_*}\sum_{l=1}^{k-1}(1-\Upsilon)^l. \tag{32}$$

Assumption 1: [2] There exist positive real scalars a_*, a^*, b_*, b^*, c_*, c^*, e_*, e^*, h_*, h^*, q_*, q^*, r_*, and r^* that guarantee:

$$\begin{aligned}&a_*^2 I \le A_i(k)A_i^T(k) \le (a^*)^2 I,\quad h_*^2 \le H_\epsilon H_\epsilon^T \le (h^*)^2 I,\\ &c_*^2 I \le C_i(k)C_i^T(k) \le (c^*)^2 I,\quad e_*^2 I \le E_i(k)E_i^T(k) \le (e^*)^2 I,\\ &b_*^2 I \le \Sigma_{j\in K_i}B_{i,j}(k)\Sigma_{j\in K_i}B_{i,j}^T(k) \le (b^*)^2 I,\\ &q_* I \le \mathcal{W}_i \le q^* I,\quad r_* I \le \mathcal{V}_i \le r^* I.\end{aligned}$$

Theorem 3: With the initial condition $\phi_* I \le \Im_i(0|0) \le \phi^* I$, Assumption 1, and the given positive scalars ϵ, α_l(l = 1-7) and λ, if $1+\alpha_5+\alpha_6 \le 1-(\alpha_3-\alpha_2)(1-\check{\psi}_i)$, $\eta_* \ge \phi_*$, $\mathbb{E}\{\mathbf{x}_i(k)\mathbf{x}_i^T(k)\} \le (X^*)^2 I$ and $\eta^* \le \phi^*$ are satisfied, then the estimation error $e_i(k|k)$ derived under the proposed algorithm is exponentially bounded in the mean square, where

$$\begin{aligned}\eta_* = &\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\varpi_*(1-\Gamma^*)^2 + (1+\alpha_1)\\ &\times\Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4-1\Big)\varpi_*\Gamma_*^2 + \Big(\Big(1+\alpha_2^{-1}+\alpha_4^{-1}\Big)\\ &\times\Big(1-\check{\psi}_i\Big)+\check{\psi}_i r_*\Big)\Gamma_*^2,\\ \eta^* = &\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\varpi^*(1-\Gamma_*)^2 + (1+\alpha_1)\\ &\times\Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4-1\Big)\varpi^*(\Gamma^*)^2 + (1+\alpha_1^{-1})\\ &\times\Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4-1\Big)\Big((1+\alpha_1)(X^*)^2\\ &+\Big(1+\alpha_1^{-1}\Big)\varpi^*\Big)(\Gamma^*)^2 + \Big(\Big(1+\alpha_2^{-1}+\alpha_4^{-1}\Big)\\ &\times\Big(1-\check{\psi}_i\Big)+\check{\psi}_i r^*\Big)(\Gamma^*)^2,\end{aligned}$$

$$\varpi_* = (1+\lambda)(h_*a_*)^2\phi_* + (1+\lambda^{-1})(h_*b_*)^2\phi_* + e_*^2 q_*,$$

$$\varpi^* = (1+\lambda)(h^*a^*)^2\phi^* + (1+\lambda^{-1})(h^*b^*)^2\phi^* + (e^*)^2q^*,$$
$$\Gamma_* = \Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\varpi_*\Big(\frac{\wp_i}{\pi^*}\Big),$$
$$\Gamma^* = \Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\varpi^*\mathbf{tr}\Big(\frac{(h^*c^*)^2}{\pi_*}\Big),$$
$$\pi_* = \Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big) + (1+\alpha_1)\Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4 - 1\Big)\Big)c_*^2\varpi_* + \Big(1-\check{\psi}_i\Big)\Big(1+\alpha_2^{-1}+\alpha_4^{-1}\Big)\xi_i^* + \check{\psi}_i r_*,$$
$$\pi^* = \Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big) + (1+\alpha_1)\Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4 - 1\Big) + \Big(\check{\psi}_i-1+\alpha_3^{-1}+\alpha_4\Big)\Big(1+\alpha_1^{-1}\Big)^2\Big)(c^*)^2\varpi^* + \Big(\check{\psi}_i-1+\alpha_3^{-1}+\alpha_4\Big)\Big(1+\alpha_1^{-1}\Big)(1+\alpha_1)(X^*)^2 \times (c^*)^2 + \Big(1-\check{\psi}_i\Big)\Big(1+\alpha_2^{-1}+\alpha_4^{-1}\Big)\xi_i^* + \check{\psi}_i r^*,$$
$$\wp_i = \lambda_{min}(H_\epsilon^T C_i^T(k)C_i(k)H_\epsilon).$$

Proof: The theorem will be proved within the following two stages.

Stage 1: We prove that $\phi_* I \le \Im_i(k|k) \le \phi^* I$ using mathematical induction method. Given the initial condition $\phi_* I \le \Im_i(0|0) \le \phi^* I$, it is obvious that the proposition holds when $k=0$; then $\phi_* I \le \Im_i(k|k) \le \phi^* I$ will be validated based on the assumption that $\phi_* I \le \Im_i(k-1|k-1) \le \phi^* I$.

To be specific, according to Eq. (17) and Assumption 1, we have:

$$\varpi_* I \le \Im_i(k|k-1) \le \varpi^* I. \tag{33}$$

Based on Lemma 1, $\mathbb{E}\{\mathbf{x}_i(k)\mathbf{x}_i^T(k)\} \le (X^*)^2 I$, and Eq. (33), it has:

$$\begin{aligned} 0 &\le \mathbb{E}\{\hat{\mathbf{x}}_i(k|k-1)\hat{\mathbf{x}}_i^T(k|k-1)\} \\ &\le (1+\alpha_1)(X^*)^2 I + \Big(1+\alpha_1^{-1}\Big)\varpi^* I. \end{aligned} \tag{34}$$

Combing Eqs. (33)-(34) with Eq. (28), then in light of Assumption 1, it can be gotten that:

$$\pi_* I \le \Pi_i(k) \le \pi^* I. \tag{35}$$

So it can be easily obtained that $(1/\pi^*)I \le \Pi_i^{-1}(k) \le (1/\pi_*)I$.

Furthermore, based on the property of trace, one has:

$$\Gamma_* I \le L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon \le \Gamma^* I. \tag{36}$$

Thus, it can be derived that:

$$(1-\Gamma^*)I \le I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon \le (1-\Gamma_*)I. \tag{37}$$

According to the expression of $L_i(k)$ given in Eq. (28) and the property of trace, we can get:

$$\begin{aligned} &\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)^2\varpi_*^2\Big(\frac{\wp_i}{(\pi^*)^2}\Big)I \le L_i(k)L_i^T(k) \\ &\le \Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)^2(\varpi^*)^2\mathbf{tr}\Big(\frac{(h^*c^*)^2}{\pi_*^2}\Big)I. \end{aligned} \tag{38}$$

Then, it is not hard to obtain that:

$$\eta_* I \le \Im_i(k|k) \le \eta^* I.$$

Thus, if the conditions $\eta_* \ge \phi_*$ and $\eta^* \le \phi^*$ hold, it can be easily concluded that:

$$\phi_* I \le \Im_i(k|k) \le \phi^* I. \tag{39}$$

Stage 2: Based on the result derived in the above stage, we then dedicate to prove that the estimation error $e_i(k|k)$ is exponentially bounded in mean square. Specifically:

Denoting $W(e_i(k|k)) = e_i^T(k|k)\Im_i^{-1}(k|k)e_i(k|k)$, then according to Eq. (39), it has:

$$(\frac{1}{\phi^* I})\|e_i(k|k)\|^2 \le W(e_i(k|k)) \le (\frac{1}{\phi_* I})\|e_i(k|k)\|^2.$$

Substituting Eq. (11) into Eq. (12), we have:

$$\begin{aligned} e_i(k|k) &= \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\Big(A_i(k-1)H_\epsilon \\ &\quad \times e_i(k-1|k-1) + g_i(k-1)\Big) - L_i(k)t_i(k), \end{aligned} \tag{40}$$

where

$$\begin{aligned} g_i(k-1) &= \Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon e_j(k-1|k-1) \\ &\quad + E_i(k-1)\omega_i(k-1), \\ t_i(k) &= \Big(\psi_i(k)-1\Big)C_i(k)H_\epsilon\mathbf{x}_i(k) + \psi_i(k)v_i(k) \\ &\quad + \Big(1-\psi_i(k)\Big)\xi_i(k). \end{aligned}$$

Then, it can be calculated that:

$$\begin{aligned} &W(e_i(k|k)) \\ &= e_i^T(k-1|k-1)H_\epsilon^T A_i^T(k-1) \\ &\quad \times \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\Im_i^{-1}(k|k) \\ &\quad \times \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)A_i(k-1)H_\epsilon \\ &\quad \times e_i(k-1|k-1) + g_i^T(k-1) \\ &\quad \times \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\Im_i^{-1}(k|k) \\ &\quad \times \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)g_i(k-1) + t_i(k)^T L_i^T(k) \\ &\quad \times \Im_i^{-1}(k|k)L_i(k)t_i(k) + e_i^T(k-1|k-1)H_\epsilon^T \\ &\quad \times A_i^T(k-1)\Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T \\ &\quad \times \Im_i^{-1}(k|k)\Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)g_i(k-1) \\ &\quad + g_i^T(k-1)\Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\Im_i^{-1}(k|k) \\ &\quad \times \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)A_i(k-1)H_\epsilon \\ &\quad \times e_i(k-1|k-1) - e_i^T(k-1|k-1)H_\epsilon^T A_i^T(k-1) \\ &\quad \times \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\Im_i^{-1}(k|k)L_i(k)t_i(k) \\ &\quad - t_i^T(k)L_i^T(k)\Im_i^{-1}(k|k)\Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big) \\ &\quad \times A_i(k-1)H_\epsilon e_i(k-1|k-1) - g_i^T(k-1) \\ &\quad \times \Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\Im_i^{-1}(k|k)L_i(k)t_i(k) \\ &\quad - t_i^T(k)L_i^T(k)\Im_i^{-1}(k|k)\Big(I - L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big) \\ &\quad \times g_i(k-1). \end{aligned} \tag{41}$$

By recurring to Lemma 1, it can be further achieved that:

$$\begin{aligned}
&W(e_i(k|k))\\
&\le (1+\alpha_5+\alpha_6)e_i^T(k-1|k-1)H_\epsilon^T A_i^T(k-1)\\
&\quad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\Im_i^{-1}(k|k)\\
&\quad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)A_i(k-1)H_\epsilon\\
&\quad\times e_i(k-1|k-1)+\Big(1+\alpha_5^{-1}+\alpha_7\Big)g_i^T(k-1)\\
&\quad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\Im_i^{-1}(k|k)\\
&\quad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)g_i(k-1)\\
&\quad+\Big(1+\alpha_6^{-1}+\alpha_7^{-1}\Big)t_i^T(k)L_i^T(k)\Im_i^{-1}(k|k)L_i(k)t_i(k). \quad (42)
\end{aligned}$$

According to Eq. (18), it is obvious that:

$$\begin{aligned}
\Im_i(k|k)\ge&\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\\
&\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\Im_i(k|k-1)\\
&\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T, \quad (43)
\end{aligned}$$

then substituting Eq. (17) into Eq. (43), we have:

$$\begin{aligned}
\Im_i(k|k)\ge&\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\\
&\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\\
&\times\Big((1+\lambda)A_i(k-1)H_\epsilon\Im_i(k-1|k-1)\\
&\times H_\epsilon^T A_i^T(k-1)+\chi_1 I\Big)\\
&\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T, \quad (44)
\end{aligned}$$

where $\chi_1=(1+\lambda^{-1})h_*^2b_*^2\phi_*+e_*^2q_*$. Given that $\Im_i(k-1|k-1)\le\phi^* I$, and pre- and post-multiplying $\Im_i^{-1}(k|k)$ by J^T and J, we can get:

$$\begin{aligned}
&H_\epsilon^T A_i^T(k-1)\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\\
&\quad\times\Im_i^{-1}(k|k)\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)\\
&\quad\times A_i(k-1)H_\epsilon\Im_i(k-1|k-1)\\
&\quad\times\Im_i^{-1}(k-1|k-1)\\
&\le\Big\{\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\Big(\Big(1+\lambda\Big)\phi^*\\
&\quad+\frac{\chi_1}{(h^*a^*)^2}\Big)\Big\}^{-1}\phi^*\Im_i^{-1}(k-1|k-1)\\
&=\chi_2\Im_i^{-1}(k-1|k-1), \quad (45)
\end{aligned}$$

where

$$\begin{aligned}
J=&\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)A_i(k-1)H_\epsilon,\\
\chi_2=&\Big\{\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\Big(\Big(1+\lambda\Big)\\
&+\frac{\chi_1}{(h^*a^*)^2\phi^*}\Big)\Big\}^{-1}. \quad (46)
\end{aligned}$$

Then, based on Eq. (44), it has:

$$\begin{aligned}
&g_i^T(k-1)\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\Im_i^{-1}(k|k)\\
&\quad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)g_i(k-1)\\
&\le\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)^{-1}g_i^T(k-1)g_i(k-1)\\
&\quad\times\Big(\Big(1+\lambda\Big)(h^*a^*)^2\phi^*+\chi_1\Big)^{-1}. \quad (47)
\end{aligned}$$

Taking the expectation on $g_i^T(k-1)g_i(k-1)$, we can get:

$$\begin{aligned}
&\mathbb{E}\{g_i^T(k-1)g_i(k-1)\}\\
&=\Big(\Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon e_j(k-1|k-1)+E_i(k-1)\\
&\quad\times\omega_i(k-1)\Big)^T\Big(\Sigma_{j\in K_i}B_{i,j}(k-1)H_\epsilon e_j(k-1|k-1)\\
&\quad+E_i(k-1)\omega_i(k-1)\Big)\\
&\le(h^*b^*)^2\phi^*+(e^*)^2q^*. \quad (48)
\end{aligned}$$

Combining Eq. (48) with Eq. (47), one has:

$$\begin{aligned}
&\mathbb{E}\Big\{g_i^T(k-1)\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)^T\Im_i^{-1}(k|k)\\
&\quad\times\Big(I-L_i(k)\Phi_{o(k)}C_i(k)H_\epsilon\Big)g_i(k-1)\Big\}\\
&\le\Big((h^*b^*)^2\phi^*+(e^*)^2q^*\Big)\Big\{\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\\
&\quad\times\Big(\Big(1+\lambda\Big)(h^*a^*)^2\phi^*+\chi_1\Big)\Big\}^{-1}\\
&=\mu_1. \quad (49)
\end{aligned}$$

Following the similar derivation process of Eq. (48), we can obtain:

$$\mathbb{E}\{t_i^T(k)t_i(k)\}\le\Big(\check{\psi}_i-1\Big)(h^*c^*)^2(X^*)^2+\Big(1-\check{\psi}_i\Big)\xi_i^*. \quad (50)$$

Thereby, it can be easily gotten that:

$$\begin{aligned}
&\mathbb{E}\Big\{t_i^T(k)L_i^T(k)\Im_i^{-1}(k|k)L_i(k)t_i(k)\Big\}\\
&\le\phi_*^{-1}\Lambda\Big\{\Big(\check{\psi}_i-1\Big)(h^*c^*)^2(X^*)^2+\Big(1-\check{\psi}_i\Big)\xi_i^*\Big\}\\
&=\mu_2. \quad (51)
\end{aligned}$$

Substituting Eqs. (33)- (34) into Eq. (28), it can be derived that:

$$L_i^T(k)L_i(k)\le\Lambda I, \quad (52)$$

where

$$\begin{aligned}
\Lambda=&\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)^2(\varpi^*)^2(h^*c^*)^2\\
&\times\Big\{\Big(1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)\Big)\varpi^*(h^*c^*)^2+(1+\alpha_1)\\
&\times\Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4-1\Big)\varpi^*(h^*c^*)^2+(1+\alpha_1^{-1})\\
&\times\Big(\check{\psi}_i+\alpha_3^{-1}+\alpha_4-1\Big)(h^*c^*)^2\Big((1+\alpha_1)(X^*)^2\\
&+\Big(1+\alpha_1^{-1}\Big)\phi^*\Big)+\Big(1+\alpha_2^{-1}+\alpha_4^{-1}\Big)\Big(1-\check{\psi}_i\Big)\xi_i^*\\
&+\check{\psi}_i r^*\Big\}^{-2}. \quad (53)
\end{aligned}$$

In view of the results presented in Eqs. (42), (45), (49), (51), (52), the conclusion below can be made.

$$\begin{aligned}
&\mathbb{E}\Big\{W(e_i(k|k))\Big\}\\
&\le(1+\alpha_5+\alpha_6)\chi_2 W(e_i(k-1|k-1))\\
&\quad+\Big(1+\alpha_5^{-1}+\alpha_7\Big)\mu_1+\Big(1+\alpha_6^{-1}+\alpha_7^{-1}\Big)\mu_2. \quad (54)
\end{aligned}$$

So, if the condition $1+\alpha_5+\alpha_6 \le 1-\Big(\alpha_3-\alpha_2\Big)\Big(1-\check{\psi}_i\Big)$ holds, one has:

$$
\begin{aligned}
&W(e_i(k|k)) - W(e_i(k-1|k-1)) \\
&\quad \le -\rho W(e_i(k-1|k-1)) + \mu,
\end{aligned} \tag{55}
$$

where

$$
\begin{aligned}
\rho &= 1-\Big(1+\lambda+\frac{\chi_1}{(h^*a^*)^2\phi^*}\Big)^{-1}, \\
\mu &= \Big(1+\alpha_5^{-1}+\alpha_7\Big)\mu_1 + \Big(1+\alpha_6^{-1}+\alpha_7^{-1}\Big)\mu_2.
\end{aligned}
$$

It is apparently that $0<\rho\le 1$ and $\mu>0$. Then, according to Lemma 3, we have:

$$
\begin{aligned}
&\mathbb{E}\{\|e_i(k|k)\|^2\} \\
&\quad \le \frac{\phi^*}{\phi_*}\mathbb{E}\{\|e_i(0|0)\|^2\}(1-\mu)^k + \rho\phi^*\sum_{l=1}^{k-1}(1-\mu)^l.
\end{aligned} \tag{56}
$$

So, the theorem is proved. □

Remark 5: We would like to emphasize that the declaimed efficient state estimators mean that the designed estimators can achieve minor estimation errors even under the influence of deception attack and limited communication bandwidth. Furthermore, given that stability is the prerequisite for system operation [37], [38], based on the proposed state estimation method which can effectively observe unavailable system state, the control strategy for the SPDSS with the aim to achieve the balanced security and stability will be studied in future.

Remark 6: In the paper, RRL-MAP-based secure state estimators are designed for SPDSSs, and some recent researches are related to our work. Focusing on DSSs, Kalman filtering and recursive filtering have been explored in [1] and [2], respectively, but the works are based on one-time-scale systems, which is different with our study. Taking the two-time-scale dynamics into account, protocol-based output feedback control for singularly perturbed fuzzy systems [8] and state estimation for singularly perturbed complex networks [21], [39] have been specifically investigated, comparing to these works which consider attack-free single-channel communication scenario, however, the state estimation method under attack-affected multichannel communication environment is explored in this study; although cyber attacks are considered while designing state estimation strategy for singularly perturbed complex networks in [12], nevertheless, the work did not employ communication protocol for preventing potential data collision. We also notice that the similar framework for the devising of recursive state estimator has been widely used [7], [22], [40], but due to the distinctive features of the considered system, differentiated measurement signals and derivation process, the unique design and analysis of the state estimation method are presented in this paper.

IV. Simulation Results

In this section, experiments based on a practical example are conducted to evaluate the performance of the proposed state estimation method. To be specific, the SPDSS formulated by Eq. (1) is applied to model a $2D$ maneuvering targets system with 3 targets (i.e., $N=3$) [1]. The sequential relationship among the 3 targets (subsystems) is depicted as $K_1=\varnothing$, $K_2=\{1\}$ and $K_3=\{1,2\}$. Each target S_i $(1\le i\le N)$ employs 4 sensors for data sensing and the corresponding WSN contains 2 nonoverlapping channels for data transmission, i.e., $n_y=4$ and $m=2$, which further imply that $U=2$. The system state of S_i is denoted as $\mathbf{x}_i(k)=col\{X_i^p(k), X_i^v(k), Y_i^p(k), Y_i^v(k)\}$ (i.e., $n_x=4$), where $X_i^p(k)$ and $X_i^v(k)$ are respectively the position coordinate and corresponding velocity of S_i along X-axe, while $Y_i^p(k)$ and $Y_i^v(k)$ are respectively the position coordinate and corresponding velocity of S_i along Y-axe.

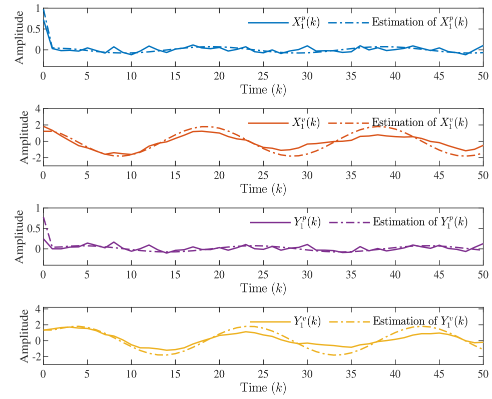

Fig. 2. Trajectories of $\mathbf{x}_1(k)$ and its estimation.

The system parameters are set as below according to [41].

$$
A_i(k)=\begin{bmatrix} 1 & \frac{sin(r_kT_k)}{r_k} & 0 & -\frac{1-cos(r_kT_k)}{r_k} \\ 0 & cos(r_kT_k) & 0 & -sin(r_kT_k) \\ 0 & \frac{1-cos(r_kT_k)}{r_k} & 1 & \frac{sin(r_kT_k)}{r_k} \\ 0 & sin(r_kT_k) & 0 & cos(r_kT_k) \end{bmatrix},
$$

$$
C_i(k)=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_i(k)=\begin{bmatrix} T_k^2/2 \\ T_k \\ T_k^2/2 \\ T_k \end{bmatrix},
$$

$$
\begin{aligned}
B_{2,1}(k) &= diag\{0, b_{2,1}^X, 0, b_{2,1}^Y\}, \quad b_{2,1}^X=0.05, b_{2,1}^Y=0.02, \\
B_{3,1}(k) &= diag\{0, b_{3,1}^X, 0, b_{3,1}^Y\}, \quad b_{3,1}^X=0.06, b_{3,1}^Y=0.07, \\
B_{3,2}(k) &= diag\{0, b_{3,2}^X, 0, b_{3,2}^Y\}, \quad b_{3,2}^X=0.05, b_{3,2}^Y=0.09,
\end{aligned}
$$

where T_k and r_k denote the time-varying sampling period and turn rate, and are set to be $T_k=0.8tanh(k)$ and $r_k=0.1+0.3tanh(k)$, respectively; $b_{2,1}^X, b_{2,1}^Y, b_{3,1}^X, b_{3,1}^Y, b_{3,2}^X, b_{3,2}^Y$ indicate different secure influence coefficients.

By further setting $n_s=2$ and $n_f=2$, the initial system states and estimation states are given as [42]:

$$
\begin{aligned}
\mathbf{x}_1(0) &= \hat{\mathbf{x}}_1(0|0) = [\sigma_{11}; 1+\sigma_{12}; \sigma_{13}; 1+\sigma_{14}], \\
\mathbf{x}_2(0) &= \hat{\mathbf{x}}_2(0|0) = [2+\sigma_{21}; 1+\sigma_{22}; 2+\sigma_{23}; 1+\sigma_{24}], \\
\mathbf{x}_3(0) &= \hat{\mathbf{x}}_3(0|0) = [2+\sigma_{31}; 2+\sigma_{32}; 2+\sigma_{33}; 2+\sigma_{34}],
\end{aligned}
$$

where $\sigma_{ij}(i=1,2,3;\ j=1,2,3,4)$ are random numbers chosen among (0,1).

The upper-bound for the estimation error covariance is initialized as $\Im_i(0|0)=I$, the SPP is set to be $\epsilon=0.05$ [39]. For the considered deception attack, we set $\xi_i^*=0.3$, $\check{\psi}_1=0.7$, $\check{\psi}_2=0.5$ and $\check{\psi}_3=0.3$. The process noise $\omega_i(k)$

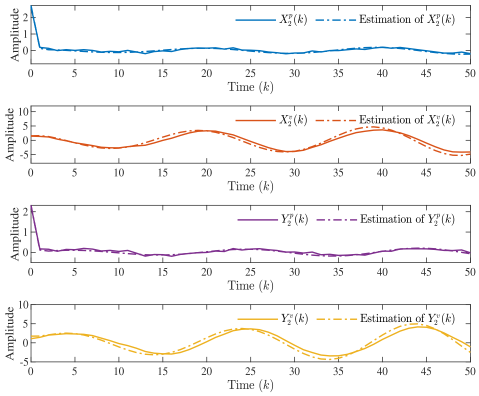

Fig. 3. Trajectories of $\mathbf{x}_2(k)$ and its estimation.

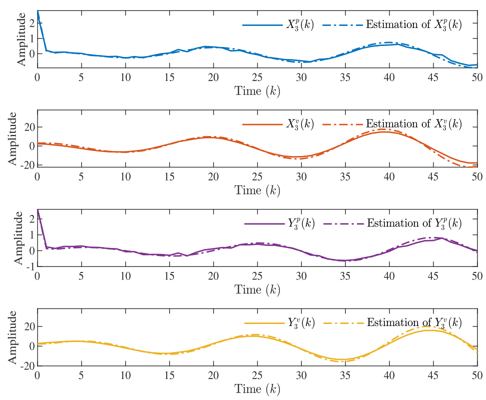

Fig. 4. Trajectories of $\mathbf{x}_3(k)$ and its estimation.

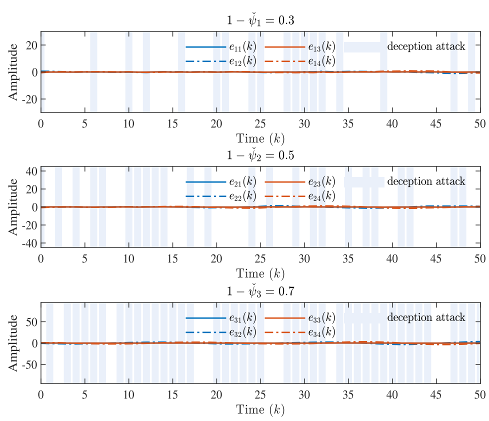

Fig. 5. The deception attack signals and estimation errors.

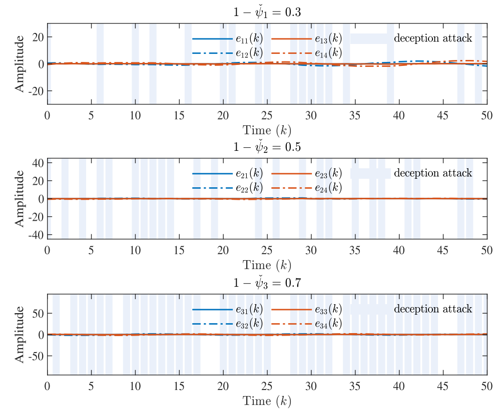

Fig. 6. The deception attack signals and estimation errors under ZOH strategy.

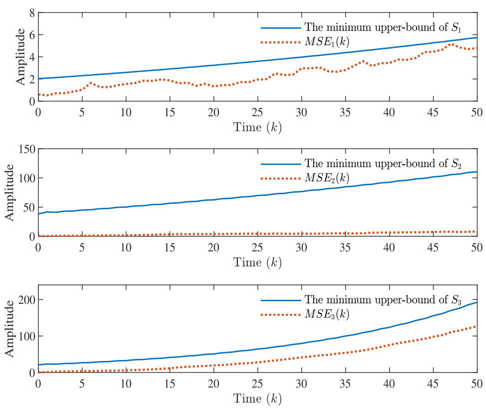

Fig. 7. $MSE_i(k)$ and the minimum upper-bounds.

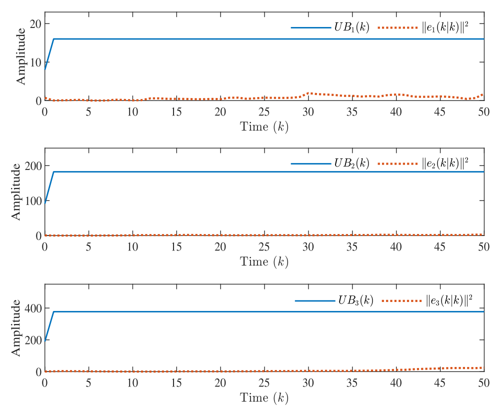

Fig. 8. $\|e_i(k|k)\|^2$ and $UB_i(k)$.

and the measurement noise $v_i(k)$ with known covariances $\mathcal{W}_i = 0.03$ and $\mathcal{V}_i = 0.03$ are generated for simulation.

Based on the above simulation settings, the time-varying estimators' gains are calculated by recurring to Algorithm 1, then the specific simulation results are presented by Figs. 2-8.

As shown in Figs. 2-4, each target's estimated positions and velocities well match its real positions and velocities, which validates the efficiency of the proposed state estimation method. The occurrence of the deception attack on each target and the corresponding estimation errors are integratedly depicted by Fig. 5. It further confirms that the designed estimators can achieve desirable performance even under deception attack scenario. The estimation errors derived under ZOH strategy is shown in Fig. 6, it can be found that ZOH method did not lead to the improvement of state estimation performance, which validates the statement presented in Remark 2.

As introduced in Theorem 2, the minimum upper-bound for each estimation error covariance can be obtained based on the designed estimator parameter, so we define the mean square error for each S_i as follows:

$$MSE_i(k) = \frac{1}{F}\sum_{f=1}^{F}\sum_{j=1}^{4}(\mathbf{x}_{i,j}^{(f)}(k) - \hat{\mathbf{x}}_{i,j}^{(f)}(k|k))^2,$$

where F is set to be 100, which means that the experiment is independently conducted 100 times, $\mathbf{x}_{i,1}(k) = X_i^p(k)$, $\mathbf{x}_{i,2}(k) = X_i^v(k)$, $\mathbf{x}_{i,3}(k) = Y_i^p(k)$, $\mathbf{x}_{i,4}(k) = Y_i^v(k)$, and $\hat{\mathbf{x}}_{i,j}(k|k)$ is the estimation of $\mathbf{x}_{i,j}(k)$. Based on the definition, the comparison between $MSE_i(k)$ and the corresponding minimum upper-bound is given in Fig. 7. As shown, $MSE_i(k)$ is always below its minimum upper-bound, which verifies the correctness of the approach for deriving the upper-bound.

Theorem 3 reveals that the estimation errors derived under the proposed algorithm can be proved to be exponentially bounded in the mean square, i.e., Eq. (56) can be achieved, if the listed conditions are satisfied. To verify this, we let $UB_i(k) = \frac{\phi^*}{\phi_*}\mathbb{E}\{\|e_i(0|0)\|^2\}(1-\mu)^k + \rho\phi^*\sum_{l=1}^{k-1}(1-\mu)^l$ and select eligible parameters as claimed in Theorem 3, then show the change curves of $UB_i(k)$ and $\|e_i(k|k)\|^2$ in Fig. 8. It can be found that each $\|e_i(k|k)\|^2$ is always smaller than corresponding $UB_i(k)$, which confirms the result presented in Theorem 3.

V. Conclusion

In this paper, for a SPDSS suffers from limited communication resource and deception attack, a distributed recursive state estimation approach has been proposed to effectively estimate system states based on available measurements. To be specific, considering that multiple nonoverlapping channels are now generally enabled in WSN, the RRL-MAP has been adopted to arrange the data transmission within each subsystem of the SPDSS. The RRL-MAP can not only realize fair and conflict-free data transmission, but also guarantee efficient channel utilization. Then, by using Bernoulli variables to depict the dynamic behavior of the considered deception attack, recursive state estimators have been designed for the attack-affected SPDSS with the RRL-MAP. Following the presented state estimation framework, upper-bounds for the estimation error covariances have been derived, and the algorithm for obtaining desirable estimators' gains has been proposed based on minimizing the achieved upper-bounds. The effectiveness of the designed algorithm has been validated by showing that the corresponding estimation errors are exponentially bounded in the mean square. By conducting experiments and analyzing simulation results, the performance of the developed state estimation method has been further demonstrated.

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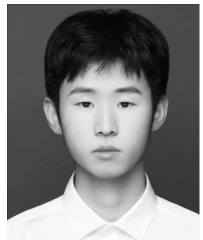

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