Secure Bipartite Consensus Control for Dynamic Event-Triggered Multi-Agent Systems Based on Co-Estimation of State and Attacks

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***Abstract*— The dynamic event-triggered (DET) bipartite consensus and attack estimation problem is investigated for multi-agent systems (MASs) under false data injection (FDI) attacks generated by exogenous systems. For the purpose of compensating for the system bias, a novel set of estimators are introduced to estimate the FDI attacks. To optimize the limited resources of the network bandwidth, a DET protocol dependant on the auxiliary variable and the local estimations is employed to regulate data transmission. By employing the designed state observer and attack observer, an novel attack tolerant event-triggered control strategy is established. Subsequently, sufficient criteria are obtained to ensure the bipartite consensus performance with l_2-l_∞ constraint. Then, by solving algebraic matrix equations and recursive linear matrix inequalities (LMI), the gains of state estimator, attack estimator and desired controller are determined, respectively. Finally, simulation examples are provided to demonstrate the validity of the derived results.**

***Note to Practitioners*—This paper discusses the secure bipartite consensus control problem for MASs subject to FDI attacks. Since the system state deviation caused by FDI attacks will bring about degradation of system performance, it is important to protect the MASs from being destroyed by designing observers to estimate the FDI attack and the system state. In view of the bandwidth constraints among the agents in MASs, optimization utilization of the network bandwidth should be taken into account seriously. Therefore, in this paper, the proposed observer-based DET controller is developed to guarantee bipartite consensus of the considered MASs while ensuring efficiency utilization of bandwidth resources. The current research provides a helpful reference for the secure bipartite consensus control of MASs with bandwidth constraints.**

***Index Terms*— Bipartite consensus control, false data injection attack, dynamic event-triggered protocol, multi-agent systems.**

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I. Introduction

OVER the years, multi-agent systems (MASs) have aroused great interests among scholars due to their strong advantages such as autonomy, distribution and coordination, and widespread applications in physical systems including, but are not limited to, mobile train lifting jack systems [1], formation systems of autonomous mobile robots (see [2], [3], [4], [5]), energy management systems of the smart grid (see [6], [7], [8], [9]), integrated hybrid energy systems [10] and multi-spacecraft systems.

Consensus control, a research hot spot of MASs, has garnered much interest [11], [12], [13], [14], [15], [16]. The consensus control indicates that all agents access the local information by the present communication topology and reach agreement asymptotically [17]. In fact, in actual communication topologies, there is not only a partnership but also a competition among agents, which can be presented by introducing negative weights [18], [19]. Many types of antagonistic interactions can be found in practical applications (e.g. biological systems, social networks and competition robotic soccer). Therefore, bipartite consensus control has emerged, where the absolute value of each agent's states heads to a final value but can be with opposing signs [20]. Currently, there are some studies investigating bipartite consensus control against MASs. Using a dynamic event-triggered (DET) scheme, the bipartite consensus issue regarding discrete-time MASs based on observers has been investigated by the authors in [21]. Reference [22] developed the finite-horizon H_∞ bipartite consensus issue for MASs using the round-robin protocol. Considering denial of service (DoS) attacks in MASs, Wang in [23] studied the bipartite consensus issue. As a special case in consensus control, the concept of bipartite consensus has emerged widely in biological systems, communication engineering and social networks. However, technical gaps remain in the research of bipartite consensus issue for MASs with resource limitation, notwithstanding the above-mentioned efforts.

Notice that practical MASs are sensitive to various disturbances, such as internal component faults (see [24], [25], [26], [27]) and external cyber attacks (see [28], [29], [30], [31], [32], [33]), which can degrade the consensus performance of the whole system [34]. False data injection (FDI) attack is one type of disruption that may strongly hide itself and infiltrate unprotected communication channels, hence biasing the state of individuals. As such, attention should be paid to the consensus control issue that takes FDI attacks into account [14], [35], [36]. Lv in [37] studied a new detection mechanism with distributed estimators in presence of FDI attacks with enough and limited energy. In [38] and [39], the event-triggered control mechanism was proposed to achieve the consensus performance for MASs with FDI attacks. It is obvious that most studies tend to model the attacks as an arbitrary bounded energy signals and ignore the variable nature of attacks. Nowadays, the correlational research on how to detect and compensate the impacts of FDI attacks are not enough, which is still an challenging issue.

It should be noted that communication resources among neighbors of MASs are limited, it is of significance to take some favorable measures for resource utilization and minimize unnecessary consumption. Thus, event-triggered protocols (e.g. static event-triggered strategy and DET approach) have been provided as a solution to address this issue [40], [41], the basic idea of which is to perform the control tasks though events. Specially, in the traditional static event-triggered strategy, the regulation tasks operate within a pre-determined sampling threshold. Under DET approach, the communication resources are allocated dynamically by including a auxiliary variable, which can regulate the transmission rates according to the system state [42]. For instance, Xu et al. in [21] developed an observer-based DET strategy for discrete-time MASs. A DET compensation controller was designed by Ju et al. in [43], where the fault information is considered. [44] designed an asynchronous event-triggered strategy for addressing the bipartite consensus issue of leader-follower MASs. Although the bipartite consensus problem of MASs have been discussed by some scholars, little attention is paid to the secure DET bipartite consensus control for MASs against FDI attacks.

Inspired by the aforementioned statements, our goal in this article is to investigate the estimation-based DET bipartite consensus issue for MASs under FDI attacks. On this basis, a joint estimator is designed to observe system state and external attack simultaneously. With this co-estimator, the system bias caused by the attack will be compensated. The following succinctly describes the primary contributions of this paper:

1)A kind of bipartite consensus control is settled for MASs under FDI attacks. Unlike [35], we use a set of estimators to estimate the FDI attacks which come from an exogenous system, thus the negative effects of the attacks can be attenuated. For reasonably using the network-band-width, a novel estimation-based dynamic event-triggered scheme is proposed to decrease the frequency of communication between agents.

2)By analysis of variances, the unbiased estimators are designed to counteract the adverse influences of FDI attacks. The secure bipartite consensus of MASs can be realized under the derived sufficient conditions. In addition, a novel secure controller with attack compensation is developed when there exists a feasible solution to the linear matrix inequality (LMI).

Notations: The n-dimensional identity matrix is denoted by I_n. $\mathbf{a}_n$ stands for an n-dimensional column vector with each of its members identical to a. diag$\{\cdots\}$ refers to a block-diagonal matrix. The mathematical exception of X is written as $E\{X\}$. sup$||Y||$ represents Y's supremum norm. $sgn(\cdot)$ is the sign function.

II. Problem Formulation and Preliminaries

A. Graph Theory

The communication network of MASs is described by a signed undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{H})$, in which $\mathcal{V} = (\upsilon_1, \upsilon_2, \ldots, \upsilon_N)$, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ and $\mathcal{H} = [g_{ij}]$ are the node set, the edge set and the weighted adjacent matrix, respectively. When $(\upsilon_i, \upsilon_j) \in \mathcal{E}$, $g_{ij} \neq 0$, it means vertices υ_i and υ_j are neighbours and they can exchange information with each other. Otherwise, $g_{ij} = 0$. In particular, $g_{ij} > 0$ and $g_{ij} < 0$ represent collaborative and competitive interaction relationship between agent i and j, respectively. Assume that $\mathcal{G}$ is free of self-loops and repetitive edges. The neighbor set of agent i is $\mathcal{N}_i = \{\upsilon_j | (\upsilon_j, \upsilon_i) \in \mathcal{E}\}$. The Laplacian matrix is $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ with $l_{ii} = \sum_{j=1, j\neq i}^{N} |g_{ij}|$ and $l_{ij} = -g_{ij}, i \neq j$.

B. System Model

Consider a team of N agents over the finite horizon $[0, T]$, where the dynamics of agent i is expressed as:

$$\begin{cases} x_{i,k+1} = A_k x_{i,k} + B_k\big(u_{i,k} + a_{i,k}\big) + D_k \omega_{i,k} \\ y_{i,k} = C_k x_{i,k} + E_k \upsilon_{i,k} \\ z_{i,k} = N_k x_{i,k}, \quad i = 1, 2, \ldots, N \end{cases} \tag{1}$$

where $x_{i,k} \in \mathbb{R}^{n_x}$ and $y_{i,k} \in \mathbb{R}^{n_y}$ are the system state and the measurement output, respectively. $u_{i,k} \in \mathbb{R}^{n_u}$ is the control law that will be designed. $a_{i,k} \in \mathbb{R}^{n_u}$ is the FDI attack signal; $\omega_{i,k} \in \mathbb{R}^{n_\omega}$ is process noise, whose means and covariance matrix are, respectively, $E\{\omega_{i,k}\} = \mu_i$ and $\sigma_i^2 I$. The value $\upsilon_{i,k} \in \mathbb{R}^{n_v}$ represents the noise in the measurement with its means being $E\{\upsilon_{i,k}\} = \nu_i$ and covariance matrix being $\psi_i^2 I$. A_k, B_k, C_k, D_k, E_k, and N_k are time-varying matrices.

To reveal the relationships of agents, we add a signed vector $\mathbf{b} = \begin{bmatrix} b_1 \ b_2 \cdots b_N \end{bmatrix}^{\mathrm{T}}$ for analytical convenience. Here, $b_i = 1$ implies that the ith agent is listed as a cooperator, while $b_i = -1$ implies that the ith agent is listed as a rival.

Definition 1 ([22]): MASs (1) is said to achieve bipartite consensus if

$$\lim_{k\to\infty} \left\| x_{i,k} - \frac{1}{N}\sum_{j=1}^{N} b_i b_j x_{j,k} \right\| = 0, \quad i = 1, 2, \ldots, N$$

C. FDI Attack Models

Following in the footsteps of [45], we assume that the hostile attacker's exogenous system is responsible for generating the FDI attack signal $a_{i,k}$:

$$\begin{cases} \eta_{i,k+1} = F_k \eta_{i,k} \\ a_{i,k} = G_k \eta_{i,k} \end{cases} \tag{2}$$

where $\eta_{i,k} \in \mathbb{R}^{n_r}$, $F_k \in \mathbb{R}^{n_r\times n_r}$ and $G_k \in \mathbb{R}^{n_r}$.

Define $\xi_{i,k} = [\, x_{i,k}^T \;\; \eta_{i,k}^T \,]^T$ and combine (2) and (1), we get

$$\begin{cases} \xi_{i,k+1} = \mathcal{A}_k \xi_{i,k} + \mathcal{B}_k u_{i,k} + \mathcal{D}_k \omega_{i,k} \\ y_{i,k} = \mathcal{C}_k \xi_{i,k} + E_k v_{i,k} \\ z_{i,k} = \mathcal{N}_k \xi_{i,k} \end{cases} \tag{3}$$

where

$$\mathcal{A}_k = \begin{bmatrix} A_k & B_k G_k \\ 0 & F_k \end{bmatrix}, \mathcal{B}_k = \begin{bmatrix} B_k \\ 0 \end{bmatrix}$$
$$\mathcal{D}_k = \begin{bmatrix} D_k^T & 0 \end{bmatrix}^T, \mathcal{C}_k = \begin{bmatrix} C_k & 0 \end{bmatrix}, \mathcal{N}_k = \begin{bmatrix} N_k & 0 \end{bmatrix}.$$

Remark 1: For the purpose of damaging the target system, the attacker can randomly select the matrices F_k and G_k. With the goal of preventing the attack signals from being detected, the matrix F_k is chosen to be a Hurwitz matrix. In this case, $a_{i,k}$ dose not approach to infinite and will be hard to detect.

D. FDI Estimator and Controller Based on DET Scheme

A set of FDI estimators will be created within this part in order to offset the signal deviation brought by FDI assaults. Additionally, an event-triggered rule is designed to relieve communication burden.

Denote the estimated FDI attack signal on agent i as

$$\hat{a}_{i,k} \triangleq G_k \hat{\eta}_{i,k} \tag{4}$$

With the virtue of proper use of the limited communication resources, event-triggered control has been widely adopted in MASs. In this article, the triggering instant t_{k+1}^i is decided by:

$$t_{k+1}^i = \inf_{k\in\mathbb{N}} \{k > t_k^i | m_{i,k}^T m_{i,k} + n_{i,k}^T n_{i,k} - \varepsilon_i y_{i,k}^T y_{i,k} > \frac{1}{\tau_i}\delta_{i,k}\} \tag{5}$$

with the gaps $m_{i,k} = y_{i,k} - y_{i,t_k^i}$ and $n_{i,k} = \hat{a}_{i,k} - \hat{a}_{i,t_k^i}$ ($k \in [t_k^i, t_{k+1}^i)$), where y_{i,t_k^i} and $\hat{a}_{i,t_k^i}$ are the measurement and the estimated FDI attack signal at t_k^i, respectively. $\varepsilon_i > 0$ and $\tau_i > 0$ are pre-determined. The internal dynamical variable $\delta_{i,k}$ satisfies

$$\begin{cases} \delta_{i,k+1} = \rho_i \delta_{i,k} - m_{i,k}^T m_{i,k} - n_{i,k}^T n_{i,k} + \varepsilon_i y_{i,k}^T y_{i,k} \\ \delta_{i,0} = \delta_0^i \end{cases} \tag{6}$$

in which $\delta_0^i \geq 0$. Furthermore, ρ_i is a prescribed constant satisfying $0 < \rho_i < 1$ and $\tau_i \geq 1/\rho_i$. At the moment the event is triggered, sensor i will transmit its measurement and estimated FDI signal attack to its neighbors at once. Clearly, $t_0^i < t_1^i < t_2^i < \cdots < t_k^i < \cdots$. The new measurement and the attack estimation will be sent to its neighbours based on (5).

To resist FDI attacks, the following observer will be developed:

$$\begin{aligned} \hat{\xi}_{i,k+1} = \mathcal{A}_k \hat{\xi}_{i,k} + \mathcal{B}_k u_{i,k} + L_{i,k}(y_{i,k} - \hat{y}_{i,k}) \\ + \mathcal{D}_k \mu_i - L_{i,k} E_k v_i \end{aligned} \tag{7}$$

where the estimated value of $\xi_{i,k}$ is $\hat{\xi}_{i,k}$, $\hat{y}_{i,k} = \mathcal{C}_k \hat{\xi}_{i,k}$ is the estimated of the measurement output and $L_{i,k}$ needs to be designed later. Based on the FDI attack estimator, the following bipartite control protocol for agent i is put forward

$$\begin{aligned} u_{i,k} &= K_k \sum_{j\in N_i} |g_{ij}|(sgn(g_{ij}) y_{j,t_k^j} - y_{i,t_k^i}) \\ &\quad - M_k \sum_{j\in N_i} |g_{ij}|(sgn(g_{ij}) \hat{a}_{j,t_k^j} - \hat{a}_{i,t_k^i}) \\ &= -K_k \sum_{j\in N_i} |g_{ij}|(sgn(g_{ij}) m_{j,k} - m_{i,k}) \\ &\quad + K_k \sum_{j\in N_i} |g_{ij}|(sgn(g_{ij}) y_{j,k} - y_{i,k}) \\ &\quad + M_k \sum_{j\in N_i} |g_{ij}|(sgn(g_{ij}) n_{j,k} - n_{i,k}) \\ &\quad - M_k \sum_{j\in N_i} |g_{ij}|(sgn(g_{ij}) \hat{a}_{j,k} - \hat{a}_{i,k}) \end{aligned} \tag{8}$$

where K_k is gain matrix of the controller.

Remark 2: To reduce the cost of communication, a unique DET method (5) is applied which is dependant on the measurement output and the estimated FDI attacks. Under the DET method, communication frequency among the agents can also be adjustable along with the change of the adaptive trigger parameters $\delta_{i,k}$. In addition, the transmitted amount of the agent state can also be controlled by adjusting the parameters τ_i, ε_i and ρ_i.

Remark 3: In practice, FDI attacks are often encountered which may do harmful to the desired performance of the MASs. For the purpose of offsetting the adverse effects of the FDI attacks, the anti-attack controller with the form of (8) is devised, which relies on the measurement output and the estimate of the FDI attacks, so as to meet the requirements of the bipartite consensus control of the MASs.

E. Estimation Error and Closed-Loop System Modeling

With the preceding description, define $e_{i,k} = \xi_{i,k} - \hat{\xi}_{i,k}$ and $e_k = \text{col}_N\{e_{i,k}\}$, then, the observation error is

$$\begin{aligned} e_{k+1} = (I_N \otimes \mathcal{A}_k - \tilde{L}_k \tilde{\mathcal{C}}_k) e_k + (I_N \otimes \mathcal{D}_k)(\omega_k - \mu) \\ - \tilde{L}_k \tilde{E}_k (v_k - \nu) \end{aligned} \tag{9}$$

where

$$\tilde{L}_k = \text{diag}\{L_{1,k}, L_{2,k}, \ldots, L_{N,k}\}$$
$$\tilde{\mathcal{C}}_k = \text{diag}\{\underbrace{\mathcal{C}_k, \ldots, \mathcal{C}_k}_{N}\}, \tilde{E}_k = \text{diag}\{\underbrace{E_k, \ldots, E_k}_{N}\}.$$

Besides, denote $\xi_k = \text{col}_N\{\xi_{i,k}\}, \eta_k = \text{col}_N\{\eta_{i,k}\}, \omega_k = \text{col}_N\{\omega_{i,k}\}$, $\mu = \text{col}_N\{\mu_i\}, v_k = \text{col}_N\{v_{i,k}\}, \nu = \text{col}_N\{v_i\}$, $m_k = \text{col}_N\{m_{i,k}\}, n_k = \text{col}_N\{n_{i,k}\}, z_k = \text{col}_N\{z_{i,k}\}$

From (3) and (8), it is not difficult to derive

$$\begin{cases} \xi_{k+1} = (I_N \otimes \mathcal{A}_k + \mathcal{L} \otimes (\mathcal{B}_k K_k \mathcal{C}_k))\xi_k \\ \quad - (\mathcal{L} \otimes \mathcal{B}_k K_k) m_k + (\mathcal{L} \otimes \mathcal{B}_k M_k) n_k \\ \quad + (\mathcal{L} \otimes \mathcal{B}_k K_k E_k) v_k + (I_N \otimes \mathcal{D}_k)\omega_k \\ \quad - (\mathcal{L} \otimes \mathcal{B}_k M_k G_k)\Xi e_k - (\mathcal{L} \otimes R_k)\xi_k \\ z_k = (I_N \otimes \mathcal{N}_k)\xi_k \end{cases} \tag{10}$$

where

$$R_k = \begin{bmatrix} 0 & \mathcal{B}_k M_k G_k \end{bmatrix}$$

$$\Xi = \begin{bmatrix} 0 & -I & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & -I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & -I \end{bmatrix}.$$

The consensus error is indicated as

$$\bar{\xi}_{i,k} = \xi_{i,k} - 1/N \sum_{j=1}^{N} b_i b_j \xi_{j,k} \tag{11}$$

which can also be expressed as $\bar{\xi}_k = (\Phi \otimes I_N)\xi_k$, in which $\Phi = I_N - 1/N\mathbf{bb}^T$.

Note that $\Phi\mathcal{L} = \mathcal{L}\Phi = \mathcal{L}$ can be derived from the properties of matrix Φ and $\mathcal{L}$. Denote $\bar{z}_k = (\Phi \otimes I_N)z_k$. Then the following (12) can be obtained from (10)

$$\begin{cases} \bar{\xi}_{k+1} = (I_N \otimes \mathcal{A}_k + \mathcal{L} \otimes (\mathcal{B}_k K_k \mathcal{C}_k))\bar{\xi}_k \\ \quad - (\mathcal{L} \otimes \mathcal{B}_k K_k)m_k + (\mathcal{L} \otimes \mathcal{B}_k M_k)n_k \\ \quad + (\mathcal{L} \otimes \mathcal{B}_k K_k E_k)v_k + (\Phi \otimes \mathcal{D}_k)\omega_k \\ \quad - (\mathcal{L} \otimes \mathcal{B}_k M_k G_k)\Xi e_k - (\mathcal{L} \otimes R_k)\bar{\xi}_k \\ \bar{z}_k = (I_N \otimes \mathcal{N}_k)\bar{\xi}_k \end{cases} \tag{12}$$

By defining the variables $\mathcal{X}_k = \begin{bmatrix} m_k^T & n_k^T \end{bmatrix}^T$ and $d_k = \begin{bmatrix} \omega_k^T & v_k^T \end{bmatrix}^T$, system (13) is obtained

$$\begin{cases} \bar{\xi}_{k+1} = \bar{A}_k \bar{\xi}_k + \bar{B}_k d_k + \bar{F}_k e_k + \bar{D}_k \mathcal{X}_k \\ \bar{z}_k = \tilde{N}_k \bar{\xi}_k \end{cases} \tag{13}$$

where

$$\bar{A}_k = I_N \otimes \mathcal{A}_k + \mathcal{L} \otimes (\mathcal{B}_k K_k \mathcal{C}_k) - \mathcal{L} \otimes R_k$$

$$\bar{D}_k = \begin{bmatrix} -(\mathcal{L} \otimes \mathcal{B}_k K_k) & (\mathcal{L} \otimes \mathcal{B}_k M_k) \end{bmatrix}$$

$$\bar{B}_k = \begin{bmatrix} \Phi \otimes \mathcal{D}_k & (\mathcal{L} \otimes \mathcal{B}_k K_k E_k) \end{bmatrix}$$

$$\bar{F}_k = -(\mathcal{L} \otimes \mathcal{B}_k M_k G_k)\Xi, \tilde{N}_k = I_N \otimes \mathcal{N}_k.$$

The purpose of this paper is to devise the FDI estimator gain L_k and controller gain K_k so that, over the finite horizon $[0, T]$, the augmented system (13) meets bipartite consensus performance for the prescribed $\gamma > 0$ with the stated l_2-l_∞ constraint

$$E\{\sup ||\bar{z}_k||^2\} \leq \sum_{k=0}^{T} \gamma^2 \{d_k^T d_k + \bar{\lambda}_{\max}\} + \gamma^2 E\{\bar{\xi}_0^T \bar{\mathcal{P}}_0 \bar{\xi}_0 + e_0^T \bar{\mathcal{Q}}_0 e_0 + \sum_{i=1}^{N} \frac{1}{\tau_i} \delta_{i,0}\} \tag{14}$$

in which $\bar{\lambda}_{\max} = \lambda_{\max}\{\tilde{\mathcal{D}}_k\}\text{trace}\{P\} + \lambda_{\max}\{\tilde{\mathcal{E}}_k\}\text{trace}\{Q\}$.

III. Main Results

A. Performance Analysis and Gain Design of the FDI Attack Estimator

Next, it will be presented that the estimation error covariance matrices (EECM) have an upper bound in least-squares sense. The FDI estimator gain L_k will be designed based on (9), which can ensure the minimization of this upper bound.

Theorem 1: For given initial condition $E\{e_0\} = 0$, the FDI attack estimator (7) is unbiased. Based on (7) and the initial condition $\mathcal{J}_0 \geq J_0$, the upper bound of EECM J_k is $\mathcal{J}_k$. Here

$$\mathcal{J}_{k+1} = (I_N \otimes \mathcal{A}_k - \tilde{L}_k \tilde{C}_k)\mathcal{J}_k(I_N \otimes \mathcal{A}_k - \tilde{L}_k \tilde{C}_k)^T + (I_N \otimes \mathcal{D}_k)P(I_N \otimes \mathcal{D}_k)^T + \tilde{L}_k \tilde{E}_k Q \tilde{E}_k^T \tilde{L}_k^T. \tag{15}$$

where

$$P = \text{diag}\left\{\sigma_1^2 I, \sigma_2^2 I, \ldots, \sigma_N^2 I\right\}$$
$$Q = \text{diag}\left\{\psi_1^2 I, \psi_2^2 I, \ldots, \psi_N^2 I\right\}$$

Moreover, the EECM can be minimized on finite horizon $[0, T]$ by designing the estimator gain $\tilde{L}_k$ as

$$\tilde{L}_k = (I_N \otimes \mathcal{A}_k)\mathcal{J}_k \tilde{C}_k^T (\tilde{C}_k \mathcal{J}_k \tilde{C}_k^T + \tilde{E}_k Q \tilde{E}_k^T)^{-1} \tag{16}$$

in which $\mathcal{J}_0$ is diagonal.

Proof: First, by taking expectation of estimation error dynamics (9), we can get:

$$\begin{aligned} E\{e_{k+1}\} =& E\{(I_N \otimes \mathcal{A}_k - \tilde{L}_k \tilde{C}_k)e_k \\ &+ (I_N \otimes \mathcal{D}_k)(\omega_k - \mu) - \tilde{L}_k \tilde{E}_k(v_k - \nu)\} \\ =& (I_N \otimes \mathcal{A}_k - \tilde{L}_k \tilde{C}_k)E\{e_k\} \end{aligned}$$

which indicates that, with $E\{e_0\} = 0$ as initial situation, the FDI attack estimation (7) is not biased.

According to (9), we can compute the covariance matrix:

$$\begin{aligned} J_{k+1} &= E\{e_{k+1}e_{k+1}^T\} \\ &= E\{[(I_N \otimes \mathcal{A}_k - \tilde{L}_k \tilde{C}_k)e_k - \tilde{L}_k \tilde{E}_k(v_k - \nu) \\ &\quad + (I_N \otimes \mathcal{D}_k)(\omega_k - \mu)][(I_N \otimes \mathcal{A}_k - \tilde{L}_k \tilde{C}_k)e_k \\ &\quad + (I_N \otimes \bar{D}_k)(\omega_k - \mu) - \tilde{L}_k \tilde{E}_k(v_k - \nu)]^T\} \\ &= (I_N \otimes \mathcal{A}_k - \tilde{L}_k \tilde{C}_k)J_k(I_N \otimes \mathcal{A}_k - \tilde{L}_k \tilde{C}_k)^T \\ &\quad + (I_N \otimes \mathcal{D}_k)P(I_N \otimes \mathcal{D}_k)^T + \tilde{L}_k \tilde{E}_k Q \tilde{E}_k^T \tilde{L}_k^T. \end{aligned}$$

It can be derived from *Lemma 1* in [46] that $J_k \leq \mathcal{J}_k$ when the initial requirement $\mathcal{J}_0 \geq J_0$ holds.

The partial derivation of trace $\mathcal{J}_k$ with respect to $\tilde{L}_k$ can subsequently be obtained

$$\begin{aligned} \frac{\partial \text{tr}(\mathcal{J}_{k+1})}{\partial \tilde{L}_k} =& -(I_N \otimes \mathcal{A}_k)\mathcal{J}_k \tilde{C}_k^T - \tilde{C}_k \mathcal{J}_k (I_N \otimes \mathcal{A}_k)^T \\ &+ \tilde{L}_k \tilde{C}_k \mathcal{J}_k \tilde{C}_k^T + \tilde{C}_k \mathcal{J}_k \tilde{C}_k^T \tilde{L}_k^T \\ &+ \tilde{L}_k \tilde{E}_k Q \tilde{E}_k^T + \tilde{E}_k Q \tilde{E}_k^T \tilde{L}_k^T. \end{aligned} \tag{17}$$

Let the partial derivation equal to zero, then, (16) can be obtained, under which the EECM is minimized.

B. Controller Design With FDI Attack Compensation

Sufficient conditions and controller design method are given in the following such that the bipartite consensus performance based on l_2-l_∞ is reached for (13).

Theorem 2: Consider MASs (1) under DET scheme (5) and FDI attacks (2). Given $\gamma > 0$ and $\varepsilon_i > 0$, matrices $\bar{\mathcal{P}}_0 > 0$ and $\bar{\mathcal{Q}}_0 > 0$, gain matrices K_k and $\tilde{L}_k$, the bipartite consensus performance can be achieved for MASs (1) with the l_2-l_∞

constraint if there exist $\mathcal{P}_k > 0$ and $\mathcal{Q}_k > 0$ (satisfying $\mathcal{P}_0 \leq \bar{\mathcal{P}}_0$ and $\mathcal{Q}_0 \leq \bar{\mathcal{Q}}_0$), and a constant $\kappa > 0$ satisfying

$$\bar{\Sigma}_k = \begin{bmatrix} \Sigma_k^{11} & * & * & * & * \\ 0 & \Sigma_k^{22} & * & * & * \\ \Sigma_k^{31} & 0 & \Sigma_k^{33} & * & * \\ \Sigma_k^{41} & 0 & \Sigma_k^{43} & \bar{\Sigma}_k^{44} & * \\ 0 & 0 & 0 & 0 & \Omega_3 \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \mathcal{P}_k & * \\ \tilde{\mathcal{N}}_k & \gamma^2 I \end{bmatrix} < 0 \tag{19}$$

Proof: Choose the following Lyapunov function candidate:

$$\mathcal{V}_k = \mathcal{V}_a(\bar{\xi}_k) + \mathcal{V}_b(e_k) + \mathcal{V}_c(\delta_k) \tag{20}$$

where

$$\mathcal{V}_a(\bar{\xi}_k) = \bar{\xi}_k^T \mathcal{P}_k \bar{\xi}_k, \mathcal{V}_b(e_k) = e_k^T \mathcal{Q}_k e_k, \mathcal{V}_c(\delta_k) = \tfrac{1}{\tau}\delta_k.$$

The difference of $\mathcal{V}_a(\bar{\xi}_k)$ along (13) is:

$$\begin{aligned} \Delta\mathcal{V}_a(\bar{\xi}_k) &= \bar{\xi}_{k+1}^T \mathcal{P}_{k+1}\bar{\xi}_{k+1} - \bar{\xi}_k^T \mathcal{P}_k \bar{\xi}_k \\ &= (\bar{A}_k\bar{\xi}_k + \bar{B}_k d_k + \bar{F}_k e_k + \bar{D}_k \mathcal{X}_k)^T \mathcal{P}_{k+1} \\ &\quad \times (\bar{A}_k\bar{\xi}_k + \bar{B}_k d_k + \bar{F}_k e_k + \bar{D}_k \mathcal{X}_k) - \bar{\xi}_k^T \mathcal{P}_k \bar{\xi}_k \\ &= \bar{\xi}_k^T (\bar{A}_k^T \mathcal{P}_{k+1}\bar{A}_k - \mathcal{P}_k)\bar{\xi}_k + 2\bar{\xi}_k^T \bar{A}_k^T \mathcal{P}_{k+1}\bar{B}_k d_k \\ &\quad + 2\bar{\xi}_k^T \bar{A}_k^T \mathcal{P}_{k+1}\bar{F}_k e_k + 2\bar{\xi}_k^T \bar{A}_k^T \mathcal{P}_{k+1}\bar{D}_k \mathcal{X}_k \\ &\quad + 2d_k^T \bar{B}_k^T \mathcal{P}_{k+1}\bar{F}_k e_k + 2d_k^T \bar{B}_k^T \mathcal{P}_{k+1}\bar{D}_k \mathcal{X}_k \\ &\quad + 2e_k^T \bar{F}_k^T \mathcal{P}_{k+1}\bar{D}_k \mathcal{X}_k + d_k^T \bar{B}_k^T \mathcal{P}_{k+1}\bar{B}_k d_k \\ &\quad + e_k^T \bar{F}_k^T \mathcal{P}_{k+1}\bar{F}_k e_k + \mathcal{X}_k^T \bar{D}_k \mathcal{P}_{k+1}\bar{D}_k \mathcal{X}_k. \end{aligned} \tag{21}$$

Notice that the FDI attack estimator (7) is unbiased with $E\{e_0\} = 0$, that is, $E\{e_k\} = 0$. Thus, it follows from (21) that

$$\begin{aligned} E\{\Delta\mathcal{V}_a(\bar{\xi}_k)\} &= E\{\bar{\xi}_k^T (\bar{A}_k^T \mathcal{P}_{k+1}\bar{A}_k - \mathcal{P}_k)\bar{\xi}_k \\ &\quad + 2\bar{\xi}_k^T \bar{A}_k^T \mathcal{P}_{k+1}\bar{B}_k d_k \\ &\quad + 2\bar{\xi}_k^T \bar{A}_k^T \mathcal{P}_{k+1}\bar{D}_k \mathcal{X}_k + 2d_k^T \bar{B}_k^T \mathcal{P}_{k+1}\bar{D}_k \mathcal{X}_k \\ &\quad + d_k^T \bar{B}_k^T \mathcal{P}_{k+1}\bar{B}_k d_k + e_k^T \bar{F}_k^T \mathcal{P}_{k+1}\bar{F}_k e_k \\ &\quad + \mathcal{X}_k^T \bar{D}_k^T \mathcal{P}_{k+1}\bar{D}_k \mathcal{X}_k\}. \end{aligned} \tag{22}$$

Similarly, on the basis of (9), one has

$$\begin{aligned} E\{\Delta\mathcal{V}_b(e_k)\} &= E\{e_{k+1}^T \mathcal{Q}_{k+1} e_{k+1} - e_k^T \mathcal{Q}_k e_k\} \\ &= E\{e_k^T (I_N \otimes \mathcal{A}_k - \tilde{L}_k\tilde{\mathcal{C}}_k)^T \mathcal{Q}_{k+1}(I_N \otimes \mathcal{A}_k \\ &\quad - \tilde{L}_k\tilde{\mathcal{C}}_k)e_k + (\omega_k - \mu)^T (I_N \otimes \mathcal{D}_k)^T \mathcal{Q}_{k+1} \\ &\quad \times (I_N \otimes \mathcal{D}_k)(\omega_k - \mu) + (v_k - \nu)^T \tilde{E}_k^T \tilde{L}_k^T \mathcal{Q}_{k+1} \\ &\quad \times \tilde{L}_k \tilde{E}_k (v_k - \nu) - e_k^T \mathcal{Q}_k e_k\} \\ &\leq e_k^T (\tilde{\mathcal{A}}_k - \mathcal{Q}_k)e_k + \lambda_{\max}\{\tilde{\mathcal{D}}_k\}\text{trace}\{P\} \\ &\quad + \lambda_{\max}\{\tilde{\mathcal{E}}_k\}\text{trace}\{Q\} \end{aligned} \tag{23}$$

where

$$\tilde{\mathcal{A}}_k = (I_N \otimes \mathcal{A}_k - \tilde{L}_k\tilde{\mathcal{C}}_k)^T \mathcal{Q}_{k+1}(I_N \otimes \mathcal{A}_k - \tilde{L}_k\tilde{\mathcal{C}}_k)$$

$$\tilde{\mathcal{D}}_k = (I_N \otimes \mathcal{D}_k)^T \mathcal{Q}_{k+1}(I_N \otimes \mathcal{D}_k)$$

$$\tilde{\mathcal{E}}_k = \tilde{E}_k^T \tilde{L}_k^T \mathcal{Q}_{k+1}\tilde{L}_k \tilde{E}_k.$$

From (6), we have

$$\begin{aligned} E\{\Delta\mathcal{V}_c(\delta_k)\} &= E\{\sum_{i=1}^N \frac{1}{\tau_i}(\delta_{i,k+1} - \delta_{i,k})\} \\ &= E\{\sum_{i=1}^N \frac{1}{\tau_i}(\rho_i\delta_{i,k} - m_{i,k}^T m_{i,k} \\ &\quad - n_{i,k}^T n_{i,k} + \varepsilon_i y_{i,k}^T y_{i,k} - \delta_{i,k})\} \\ &= E\{\sum_{i=1}^N \frac{\rho_i - 1}{\tau_i}\delta_{i,k} + \bar{\xi}_k^T \tilde{\mathcal{C}}_k^T \Omega_1 \tilde{\mathcal{C}}_k \bar{\xi}_k - \mathcal{X}_k^T \Omega_2 \mathcal{X}_k \\ &\quad + 2\bar{\xi}_k^T \tilde{\mathcal{C}}_k^T \Omega_1 \tilde{E}_k v_k + v_k^T \tilde{E}_k^T \Omega_1 \tilde{E}_k v_k\} \end{aligned} \tag{24}$$

where

$$\Omega_1 = \text{diag}\left\{\frac{\varepsilon_1}{\tau_1}I, \ldots, \frac{\varepsilon_N}{\tau_N}I\right\},$$

$$\Omega_2 = \text{diag}\left\{\underbrace{\frac{1}{\tau_1}I, \frac{1}{\tau_1}I, \ldots, \frac{1}{\tau_N}I, \frac{1}{\tau_N}I}_{2N}\right\}.$$

Based on the definition of d_k, $\Delta\mathcal{V}_c$ can be further written as

$$\begin{aligned} E\{\Delta\mathcal{V}_c(\delta_k)\} &= E\{\sum_{i=1}^N \frac{\rho_i - 1}{\tau_i}\delta_{i,k} + \bar{\xi}_k^T \tilde{\mathcal{C}}_k^T \Omega_1 \tilde{\mathcal{C}}_k \bar{\xi}_k - \mathcal{X}_k^T \Omega_2 \mathcal{X}_k \\ &\quad + 2\bar{\xi}_k^T \tilde{\mathcal{C}}_k^T \Omega_1 \mathcal{E}_k d_k + d_k^T \mathcal{E}_k^T \Omega_1 \mathcal{E}_k d_k\} \end{aligned} \tag{25}$$

where $\mathcal{E}_k = \begin{bmatrix} 0 & \tilde{E}_k \end{bmatrix}$.

It can be seen that (5) implies

$$\begin{aligned} &\sum_{i=1}^N \kappa(m_{i,k}^T m_{i,k} + n_{i,k}^T n_{i,k} - \frac{1}{\tau_i}\delta_{i,k} - \varepsilon_i y_{i,k}^T y_{i,k}) \\ &\leq 0. \end{aligned} \tag{26}$$

Denote $\theta_k = [\,\bar{\xi}_k^T \; e_k^T \; \mathcal{X}_k^T \; d_k^T \; \bar{\delta}_k^T\,]^T$ and $\bar{\delta}_k = [\delta_{1,k}^{\frac{1}{2}} \cdots \delta_{N,k}^{\frac{1}{2}}]^T$. Substituting (22), (23), (25) and (26) into (20), it yields that

$$\begin{aligned} E\{\Delta\mathcal{V}_k\} &\leq E\{\bar{\xi}_k^T (\bar{A}_k^T \mathcal{P}_{k+1}\bar{A}_k + \tilde{\mathcal{C}}_k^T \bar{\Omega}_1 \tilde{\mathcal{C}}_k - \mathcal{P}_k)\bar{\xi}_k \\ &\quad + 2\bar{\xi}_k^T \bar{A}_k^T \mathcal{P}_{k+1}\bar{B}_k d_k + 2\bar{\xi}_k^T \bar{A}_k^T \mathcal{P}_{k+1}\bar{D}_k \mathcal{X}_k \\ &\quad + 2d_k^T \bar{B}_k^T \mathcal{P}_{k+1}\bar{D}_k \mathcal{X}_k + d_k^T \bar{B}_k^T \mathcal{P}_{k+1}\bar{B}_k d_k \\ &\quad + e_k^T \bar{F}_k^T \mathcal{P}_{k+1}\bar{F}_k e_k^T + \mathcal{X}_k^T \bar{D}_k^T \mathcal{P}_{k+1}\bar{D}_k \mathcal{X}_k \\ &\quad + e_k^T (\tilde{\mathcal{A}}_k - \mathcal{Q}_k)e_k + \sum_{i=1}^N \frac{\rho_i + \kappa - 1}{\tau_i}\delta_{i,k} \\ &\quad - \mathcal{X}_k^T \bar{\Omega}_2 \mathcal{X}_k + 2\bar{\xi}_k^T \tilde{\mathcal{C}}_k^T \bar{\Omega}_1 \mathcal{E}_k d_k + d_k^T \mathcal{E}_k^T \bar{\Omega}_1 \mathcal{E}_k d_k \\ &\quad + \lambda_{\max}\{\tilde{\mathcal{D}}_k\}\text{trace}\{P\} + \lambda_{\max}\{\tilde{\mathcal{E}}_k\}\text{trace}\{Q\}\} \\ &= E\{\theta_k^T \Sigma_k \theta_k + \lambda_{\max}\{\tilde{\mathcal{D}}_k\}\text{trace}\{P\} \\ &\quad + \lambda_{\max}\{\tilde{\mathcal{E}}_k\}\text{trace}\{Q\}\} \end{aligned} \tag{27}$$

where

$$\Sigma_k = \begin{bmatrix} \Sigma_k^{11} & * & * & * & * \\ 0 & \Sigma_k^{22} & * & * & * \\ \Sigma_k^{31} & 0 & \Sigma_k^{33} & * & * \\ \Sigma_k^{41} & 0 & \Sigma_k^{43} & \Sigma_k^{44} & * \\ 0 & 0 & 0 & 0 & \Omega_3 \end{bmatrix}$$

$$\Sigma_k^{11} = \bar{A}_k^T \mathcal{P}_{k+1}\bar{A}_k + \tilde{\mathcal{C}}_k{}^T \bar{\Omega}_1 \tilde{\mathcal{C}}_k - \mathcal{P}_k$$

$$\begin{aligned}
\Sigma_k^{22} &= \bar{F}_k^T \mathcal{P}_{k+1} \bar{F}_k + \bar{\mathcal{A}}_k - \mathcal{Q}_k \\
\Sigma_k^{33} &= \bar{D}_k^T \mathcal{P}_{k+1} \bar{D}_k - \bar{\Omega}_2 \\
\Sigma_k^{44} &= \bar{B}_k^T \mathcal{P}_{k+1} \bar{B}_k + \mathcal{E}_k^T \bar{\Omega}_1 \mathcal{E}_k \\
\Sigma_k^{31} &= \bar{A}_k^T \mathcal{P}_{k+1} \bar{D}_k,\ \Sigma_k^{43} = \bar{D}_k^T \mathcal{P}_{k+1} \bar{B}_k \\
\Sigma_k^{41} &= \bar{A}_k^T \mathcal{P}_{k+1} \bar{B}_k + \tilde{\mathcal{C}}_k^T \bar{\Omega}_1 \mathcal{E}_k \\
\bar{\Omega}_1 &= \operatorname{diag}\left\{ \varepsilon_1 (\frac{1}{\tau_1} + \kappa) I, \ldots, \varepsilon_N (\frac{1}{\tau_N} + \kappa) I \right\} \\
\bar{\Omega}_2 &= \kappa I + \operatorname{diag}\left\{ \underbrace{\frac{1}{\tau_1} I, \frac{1}{\tau_1} I, \ldots, \frac{1}{\tau_N} I, \frac{1}{\tau_N} I}_{2N} \right\} \\
\Omega_3 &= \operatorname{diag}\left\{ \frac{\rho_1 + \kappa - 1}{\tau_1}, \ldots, \frac{\rho_N + \kappa - 1}{\tau_N} \right\}.
\end{aligned}$$

In what follows, we will further process the formula to investigate the consensus performance under the l_2-l_∞ restriction. According to $\bar{z}_k$, we have

$$\begin{aligned}
\bar{z}_k^T \bar{z}_k &= \bar{\xi}_k^T \tilde{N}_k^T \tilde{N}_k \bar{\xi}_k \\
&\leq \gamma^2 \bar{\xi}_k^T \mathcal{P}_k \bar{\xi}_k \\
&\leq \gamma^2 \left(\bar{\xi}_k^T \mathcal{P}_k \bar{\xi}_k + e_k^T \mathcal{Q}_k e_k + \frac{1}{\tau} \delta_k \right) \\
&\leq \mathcal{V}_k.
\end{aligned} \tag{28}$$

Then, according to (27), we have

$$\begin{aligned}
&E\{\mathcal{V}_k - \mathcal{V}_0 - \gamma^2 \sum_{k=0}^{T} (d_k^T d_k + \bar{\lambda}_{\max})\} \\
&= E\{\sum_{k=0}^{T} \Delta \mathcal{V}_k - \gamma^2 \sum_{k=0}^{T} (d_k^T d_k + \bar{\lambda}_{\max})\} \\
&\leq \sum_{k=0}^{T} E\{\theta_k^T \bar{\Sigma}_k \theta_k\} \\
&\leq 0
\end{aligned} \tag{29}$$

where

$$\bar{\Sigma}_k = \begin{bmatrix} \Sigma_k^{11} & * & * & * & * \\ 0 & \Sigma_k^{22} & * & * & * \\ \Sigma_k^{31} & 0 & \Sigma_k^{33} & * & * \\ \Sigma_k^{41} & 0 & \Sigma_k^{43} & \bar{\Sigma}_k^{44} & * \\ 0 & 0 & 0 & 0 & \Omega_3 \end{bmatrix}$$

$$\bar{\Sigma}_k^{44} = \bar{B}_k^T \mathcal{P}_{k+1} \bar{B}_k + \mathcal{E}_k^T \bar{\Omega}_1 \mathcal{E}_k - \gamma^2 I.$$

The above formula can be further rewritten as

$$E\{\mathcal{V}_k\} \leq \sum_{k=0}^{T} \gamma^2 \{d_k^T d_k + \bar{\lambda}_{\max}\} + E\{\mathcal{V}_0\}. \tag{30}$$

Taking the conditions $\mathcal{P}_0 \leq \bar{\mathcal{P}}_0$ and $\mathcal{Q}_0 \leq \bar{\mathcal{Q}}_0$ as well as (28) and (30) into consideration, it follows that

$$\begin{aligned}
E\{\sup ||\bar{z}_k||^2\} \leq \gamma^2 E\{\bar{\xi}_0^T \bar{\mathcal{P}}_0 \bar{\xi}_0 + e_0^T \bar{\mathcal{Q}}_0 e_0 + \sum_{i=1}^{N} \frac{1}{\tau_i} \delta_{i,0}\} \\
+ \sum_{k=0}^{T} \gamma^2 \{d_k^T d_k + \bar{\lambda}_{\max}\}.
\end{aligned} \tag{31}$$

which indicates that the bipartite consensus performance with constraint (14) is obtained.

In what follows, the controller gains will be obtained based on the results in Theorem 2.

Theorem 3: For given parameters ρ_i, τ_i, ε_i, positive scalar γ, two weighted matrices $\bar{\mathcal{P}}_0$ and $\bar{\mathcal{Q}}_0$, if there exist $\bar{\mathcal{P}}_k > 0$ and $\bar{\mathcal{Q}}_k > 0$, matrices $\bar{\mathcal{K}}_k$, $\bar{\mathcal{Z}}_{11k}$, $\bar{\mathcal{Z}}_{12k}$ and $\bar{\mathcal{Z}}_{22k}$ such that the subsequent LMI:

$$\tilde{\Xi}_k = \begin{bmatrix} \Xi_k^1 & * \\ \tilde{\Xi}_k^2 & \tilde{\Xi}_k^3 \end{bmatrix} < 0 \tag{32}$$

$$\begin{bmatrix} \mathcal{P}_k & * \\ \tilde{\mathcal{N}}_k & \gamma^2 I \end{bmatrix} < 0 \tag{33}$$

hold, where

$$\Xi_k^1 = \begin{bmatrix} \Xi_k^{11} & * & * & * & * \\ 0 & -\mathcal{Q}_k & * & * & * \\ 0 & 0 & -\bar{\Omega}_2 & * & * \\ \Xi_k^{41} & 0 & 0 & \Xi_k^{44} & * \\ 0 & 0 & 0 & 0 & \Omega_3 \end{bmatrix}$$

$$\tilde{\Xi}_k^2 = \begin{bmatrix} \bar{\Xi}_k^{61} & 0 & \bar{\Xi}_k^{63} & \bar{\Xi}_k^{64} & 0 \\ 0 & \bar{F}_k & 0 & 0 & 0 \\ 0 & \Xi_k^{82} & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Xi}_k^3 = \begin{bmatrix} \mathcal{P}_{k+1} - \mathcal{Z}_k - \mathcal{Z}_k^T & * & * \\ 0 & -\mathcal{P}_{k+1} & * \\ 0 & 0 & -\mathcal{Q}_{k+1} \end{bmatrix}.$$

Then system (13) achieves the bipartite consensus performance with constraint (14). In this case, the controller gains can be acquired by $K_k = \mathcal{Z}_{11k}^{-1} \bar{\mathcal{K}}_k$.

Proof: First, by using the Schur complement lemma, (18) can be rewritten as follows:

$$\Xi_k = \begin{bmatrix} \Xi_k^1 & * \\ \Xi_k^2 & \Xi_k^3 \end{bmatrix} < 0 \tag{34}$$

where

$$\Xi_k^1 = \begin{bmatrix} \Xi_k^{11} & * & * & * & * \\ 0 & -\mathcal{Q}_k & * & * & * \\ 0 & 0 & -\bar{\Omega}_2 & * & * \\ \Xi_k^{41} & 0 & 0 & \Xi_k^{44} & * \\ 0 & 0 & 0 & 0 & \Omega_3 \end{bmatrix}$$

$$\Xi_k^2 = \begin{bmatrix} \bar{A}_k & 0 & \bar{D}_k & \bar{B}_k & 0 \\ 0 & \bar{F}_k & 0 & 0 & 0 \\ 0 & \Xi_k^{82} & 0 & 0 & 0 \end{bmatrix}$$

$$\Xi_k^3 = \begin{bmatrix} -\mathcal{P}_{k+1}^{-1} & * & * \\ 0 & -\mathcal{P}_{k+1}^{-1} & * \\ 0 & 0 & -\mathcal{Q}_{k+1}^{-1} \end{bmatrix}$$

$$\begin{aligned}
\Xi_k^{11} &= \tilde{\mathcal{C}}_k^T \bar{\Omega}_1 \tilde{\mathcal{C}}_k - \mathcal{P}_k,\ \Xi_k^{41} = \tilde{\mathcal{C}}_k^T \bar{\Omega}_1 \mathcal{E}_k \\
\Xi_k^{44} &= \mathcal{E}_k^T \bar{\Omega}_1 \mathcal{E}_k - \gamma^2 I,\ \Xi_k^{82} = I_N \otimes \mathcal{A}_k - \tilde{L}_k \tilde{\mathcal{C}}_k.
\end{aligned}$$

Next, pre- and post-multiplying inequality (34) with matrix $\operatorname{diag}\{I, I, I, I, I, \mathcal{Z}_k, \mathcal{P}_{k+1}, \mathcal{Q}_{k+1}\}$ and its transposition, we can get

$$\bar{\Xi}_k = \begin{bmatrix} \Xi_k^1 & * \\ \bar{\Xi}_k^2 & \bar{\Xi}_k^3 \end{bmatrix} < 0 \tag{35}$$

where

$$
\begin{aligned}
\bar{\Xi}_k^2 &= \begin{bmatrix} \bar{\Xi}_k^{61} & 0 & \bar{\Xi}_k^{63} & \bar{\Xi}_k^{64} & 0 \\ 0 & \bar{F}_k & 0 & 0 & 0 \\ 0 & \Xi_k^{82} & 0 & 0 & 0 \end{bmatrix} \\
\bar{\Xi}_k^3 &= \begin{bmatrix} -\mathcal{Z}_k \mathcal{P}_{k+1}^{-1} \mathcal{Z}_k^T & * & * \\ 0 & -\mathcal{P}_{k+1} & * \\ 0 & 0 & -\mathcal{Q}_{k+1} \end{bmatrix} \\
\bar{\Xi}_k^{61} &= I_N \otimes \mathcal{Z}_{1k} \mathcal{W}_k \mathcal{A}_k + \mathcal{L} \otimes \mathcal{K}_k \mathcal{C}_k - \mathcal{L} \otimes \mathcal{Z}_{1k} \mathcal{W}_k R_k \\
\bar{\Xi}_k^{64} &= \left[\Phi \otimes \mathcal{Z}_{1k} \mathcal{W}_k \mathcal{D}_k \ (\mathcal{L} \otimes \mathcal{K}_k E_k) \right] \\
\bar{\Xi}_k^{63} &= \left[-\mathcal{L} \otimes \mathcal{K}_k \ \mathcal{L} \otimes \mathcal{Z}_{1k} \mathcal{W}_k \mathcal{B}_k M_k \right] \\
\mathcal{Z}_k &= I_N \otimes \mathcal{Z}_{1k} \mathcal{W}_k \\
\mathcal{Z}_{1k} &= \begin{bmatrix} \mathcal{Z}_{11k} & \mathcal{Z}_{12k} \\ 0 & \mathcal{Z}_{22k} \end{bmatrix} \\
\mathcal{W}_k &= \left[\mathcal{B}_k (\mathcal{B}_k^T \mathcal{B}_k)^{-1} \ (\mathcal{B}_k^T)^{\perp} \right]^T \\
\mathcal{K}_k &= \left[\bar{\mathcal{K}}_k^T \ 0 \right]^T = \mathcal{Z}_{1k} \mathcal{W}_k \mathcal{B}_k K_k.
\end{aligned}
$$

According to the following inequality

$$
-\mathcal{Z}_k \mathcal{P}_{k+1}^{-1} \mathcal{Z}_k^T \leq \mathcal{P}_{k+1} - \mathcal{Z}_k - \mathcal{Z}_k^T
$$

one can deduce that the (35) can be ensured by the following one

$$
\tilde{\Xi}_k = \begin{bmatrix} \Xi_k^1 & * \\ \bar{\Xi}_k^2 & \tilde{\Xi}_k^3 \end{bmatrix} < 0 \tag{36}
$$

Remark 4: By now, Theorem 1 and Theorem 3 have provided the estimator and the anti-attack controller design method. We can observe from the design process that the bipartite consensus performance of the MASs is influenced by the FDI attack, the system parameters and the DET mechanism.

Remark 5: It is important to highlight that the innovative DET strategy (5) is characterized by its adaptive threshold parameter, which, rather than being static, is dynamically tuned following the principles of the dynamic law (6). This flexibility allows for more responsive and precise control in varying conditions.

Remark 6: In contrast to the work in [20] and [21], where the network is assumed to be secure, the proposed method considers the adverse effects of the FDI attacks, which is more challenging. Unlike the DET strategy in [23], the DET method in this paper involves the measurement output and the estimated FDI attacks. This proactive approach allows network control systems to respond promptly to attacks, ensuring the safety of MASs. Thus, the designed secure DET bipartite control method has better anti-attack ability against FDI attacks and better utilizaiton of the network bandwidth.

IV. Simulation Results

In this section, the effectiveness of the developed bipartite consensus control strategy is applied on a numerical example and unmanned aerial vehicles (UAVs).

Example 1: To verify the effectiveness of the developed approach in this paper, a cooperation-competition MAS with communication topology shown in Fig. 1 will be employed. It is assumed that agents 1 and 2 are competitors, whereas the remaining agents are cooperators. Here is the undirected Laplacian matrix $\mathcal{L}$:

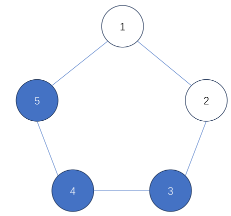

Fig. 1. Communication topology among five agents.

TABLE I

Controller Parameters

k	1	2	3	4	5	6
K_k	0.1037	0.0717	0.0436	0.0316	0.0185	0.0280
k	7	8	9	10	11	12
K_k	0.0333	0.0526	-0.2028	0.0926	0.1270	0.1457
k	13	14	15	16	17	$\cdots$
K_k	0.1420	0.0940	0.0520	0.0125	0.0542	$\cdots$

$$
\mathcal{L} = \begin{bmatrix} 1.5 & -0.5 & 0.5 & -0.5 & 0 \\ -0.5 & 1.5 & -0.5 & 0 & -0.5 \\ 0.5 & -0.5 & 2 & -0.5 & 0.5 \\ -0.5 & 0 & -0.5 & 1.5 & -0.5 \\ 0 & -0.5 & 0.5 & -0.5 & 1.5 \end{bmatrix}.
$$

The parameters of system (1) are set as:

$$
\begin{aligned}
A_k &= \begin{bmatrix} 0.35 + 0.05\cos(0.4k) & -0.10 \\ -0.10 & -0.73 - 0.1\cos(0.5k) \end{bmatrix} \\
B_k &= \begin{bmatrix} 0.1 \\ 0.25 \end{bmatrix}, D_k = \begin{bmatrix} 0.2 \\ 0.08 \end{bmatrix}, F_k = \begin{bmatrix} 0.1 \\ -0.5 \end{bmatrix} \\
C_k &= \left[1.05 \ 0.1 \right], E_k = \left[0.1 \ 0.15 \right] \\
N_k &= \left[0.2 \ 0.2 \right], M_k = 1 \\
G_k &= \begin{bmatrix} 0.8 & 0.2 \\ -0.6 & 0.8 \end{bmatrix}.
\end{aligned}
$$

The finite horizon is set at [0,45] in the simulation. Besides, we select the initial states as $x_{1,0} = [1.27 \quad 1.21]^T$, $x_{2,0} = [-1.3 \quad -1.31]^T$, $x_{3,0} = [1.17 \quad 1.30]^T$, $x_{4,0} = [-1.20 \quad -1.17]^T$, $x_{5,0} = [1.11 \quad 3]^T$, $\eta_{1,1} = [4 \quad 3]^T$, $\eta_{2,1} = [-4 \quad -3]^T$, $\eta_{3,1} = [4 \quad 3]^T$, $\eta_{4,1} = [-1 \quad -2]^T$ and $\eta_{5,1} = [3 \quad 3]^T$. The values of $\mu_i = 0.1$ and $\nu_i = 0.1$ are the means. $\sigma_i^2 = 0.1$ and $\psi_i^2 = 0.4$ are selected as the covariances. In (5) and (6), the initials and the dynamic variables are given by $\varepsilon_1 = \varepsilon_4 = 0.5$, $\varepsilon_2 = \varepsilon_5 = 0.6$, $\varepsilon_3 = 0.7$, $\delta_0^1 = \delta_0^4 = \delta_0^5 = 1$ and $\delta_0^2 = \delta_0^3 = 2$. The other parameters are chosen as $\tau_1 = 400$, $\tau_2 = 400$, $\tau_3 = 22$, $\tau_4 = 150$, $\tau_5 = 10$, and $\rho_1 = 0.06$, $\rho_2 = 0.06$, $\rho_3 = 0.02$, $\rho_4 = 0.5$ and $\rho_5 = 0.7$. Moreover, TABLE I displays the intended controller gains.

The outcomes of the simulation are displayed in Figs. 2-8. Specifically, Fig.2 and Fig.3 show the state dynamics of each agent for MAS (1) employing the bipartite consensus controller. It can be ascertained that the suggested compensation strategy makes sense. Furthermore, Figs. 4-6 depict the attacks and their estimation signals, which show that the intended attack estimating approach is useful and efficient. Since the attacks and its estimations on agent 4 and 5 can be drawn

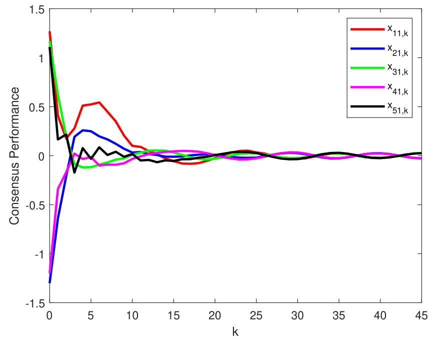

Fig. 2. Trajectories of the agent state $x_{i1,k}$ under FDI attacks.

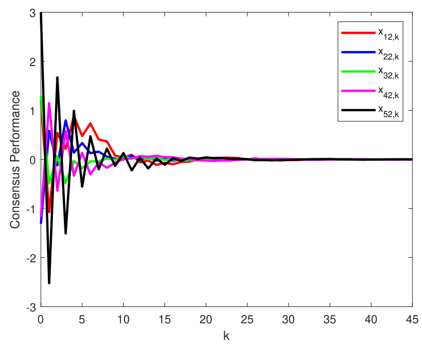

Fig. 3. Trajectories of the agent state $x_{i2,k}$ under FDI attacks.

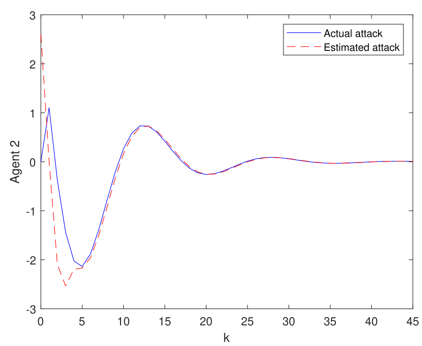

Fig. 4. Actual attack on Agent 2 and its estimate.

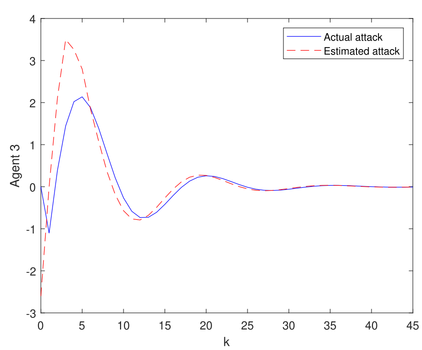

Fig. 5. Actual attack on Agent 3 and its estimate.

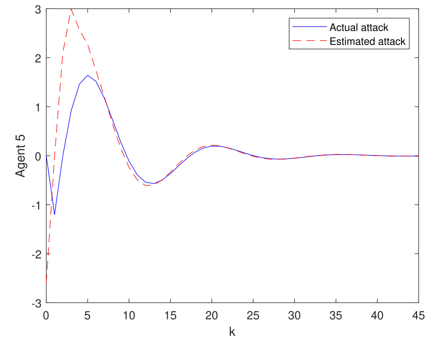

Fig. 6. Actual attack on Agent 5 and its estimate.

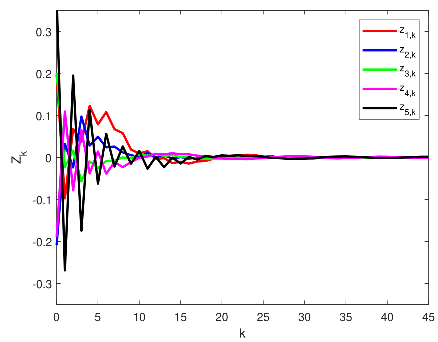

Fig. 7. Dynamics of the controlled outputs with compensation control.

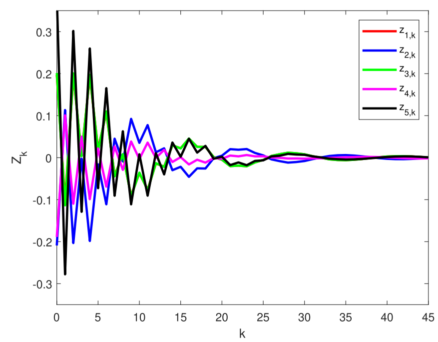

Fig. 8. Dynamics of the controlled outputs without compensation control.

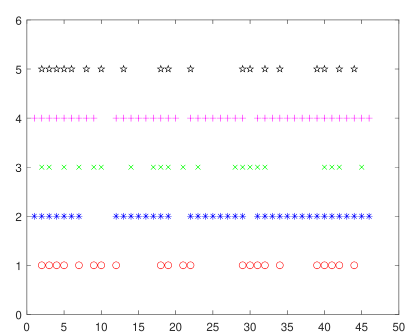

Fig. 9. Triggered instants of five agents.

similarly, we omit the figures here for paper limitation. The controlled output for the agents with and without control compensation mechanism are depicted in Fig. 7 and Fig. 8, respectively. By comparing the two figures of dynamics of the controlled outputs, it is not hard to reach that the proposed compensation method has better performance. The proposed controller with control compensation mechanism has higher ability in anti-interference and faster convergence speed. The event-triggered release moments of each agent are exhibited in Fig. 9, from which one can draw that the limited network resources can be efficiently saved by the suggested DET strategy.

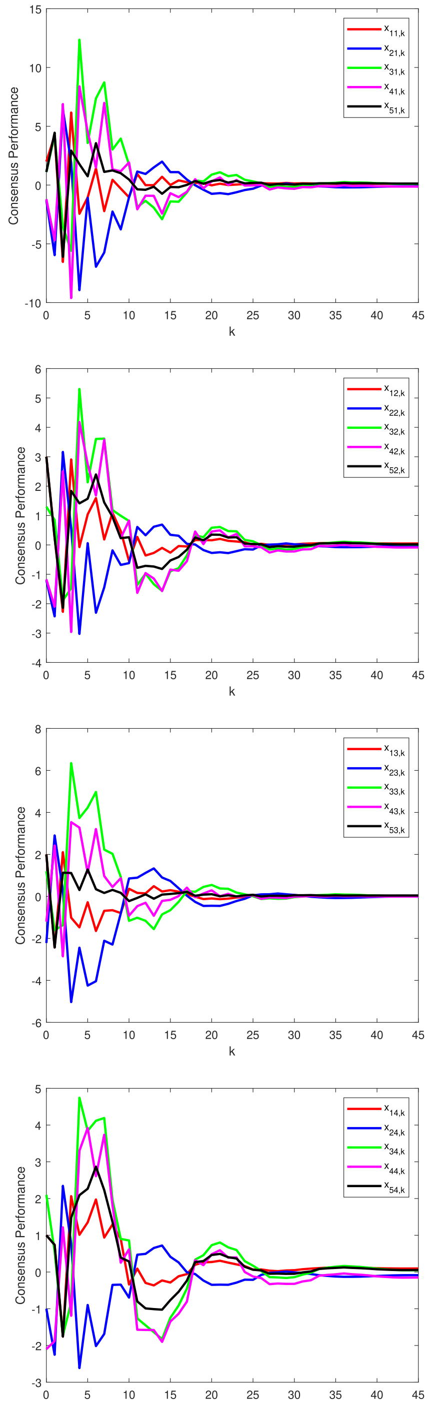

Fig. 10. State trajectories of the agents in Example 2.

Example 2: In this example, the MASs under consideration consisting of 5 YF-22 research UAVs [47] whose longitudinal dynamics satisfy (1) with

$$A_k = \begin{bmatrix} -0.284 & -0.296 & 2.420 & 0.9912 \\ 0 & -0.4117 & 0.843 & 0.272 \\ 0 & -0.338 & -0.826 & -0.195 \\ 0 & 0 & 0.6 & 0 \end{bmatrix}$$

$$B_k = \begin{bmatrix} 0.2168 \\ 0.544 \\ -0.3908 \\ 0.6 \end{bmatrix}, D_k = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.8 \\ 0.8 \end{bmatrix}, F_k = \begin{bmatrix} 0.8 & 0.2 \\ -0.6 & 0.8 \end{bmatrix}$$

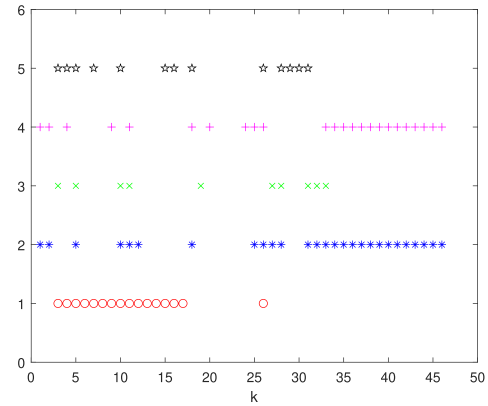

Fig. 11. Triggered instants of UAVs.

$$C_k = \begin{bmatrix} 0.4 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, E_k = \begin{bmatrix} 0.1 \\ 0.15 \end{bmatrix}$$
$$G_k = \begin{bmatrix} 0.1 & -0.15 \end{bmatrix}.$$

where $x_i(k) = [x_{i1}(k), x_{i2}(k), x_{i3}(k), x_{i4}(k)]^T$ and $x_{i1}(k), x_{i2}(k), x_{i3}(k), x_{i4}(k)$ represent, respectively, the speed, the attack angle, the pitch rate, the pitch angle. The topology of the MASs is shown in Fig. 1. According to Theorem 3, by choosing $\varepsilon_1 = \varepsilon_5 = 0.5$, $\varepsilon_2 = \varepsilon_4 = 0.6$, $\varepsilon_3 = 0.7$, $\tau_1 = 400$, $\tau_2 = 40$, $\tau_3 = 22$, $\tau_4 = 150$, $\tau_5 = 1000$, and $\rho_1 = 0.06$, $\rho_2 = 0.6$, $\rho_3 = 0.02$, $\rho_4 = 0.07$ and $\rho_5 = 0.7$, one obtains $K_k = \begin{bmatrix} -0.0198 & -0.2833 \end{bmatrix}$. With the obtained control scheme, the trajectories of the agents are depicted in Fig. 10. The triggered instants are illustated in Fig.11, from which one can see dynamically adjust the bandwidth resource usage and avoid the unnecessary transmissions. Therefore, the frequency for control updates can be decreased effectively. From the simulation results, one can observe that by the developed controller, the effect of the FID attacks can be well dealt with and the distributed bipartite consensus is realized successfully.

V. Conclusion

In this study, we have investigated the bipartite consensus control issue for DTE MASs with FDI attacks. A set of estimators have been developed to estimate the attacks and each agents' state. For lightening the burden of communication networks, a DET strategy has been developed where each agent is allowed to broadcast its estimates when the triggering function is satisfied. With the methods of variance analysis and the Lyapunov stability theory, the expected estimator and controller gains have been designed. The effectiveness of the developed appoach has also been confirmed by the simulation results. In the future, we will design the mode-free fault tolerant control for MASs with complex attacks and DET scheme. Another interesting direction of our future work is the predictive control for MASs with DoS attacks and signal compensation.

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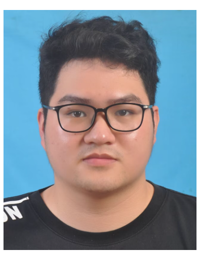

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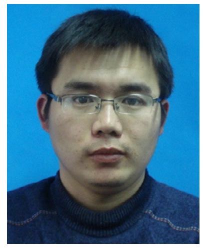

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