Protocol-Based Distributed Security Fusion Estimation for Networked Systems With Unknown Bounded Noise Under Quantization

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***Abstract*—In this article, a novel distributed fusion estimation approach is proposed for time-varying systems with unknown bounded noises subject to bandwidth-constrained networks and denial-of-service (DoS) attacks, where the round-robin scheduling protocol and the quantization scheme are, respectively, employed to ease the burden of networks. It is assumed that the network connecting sensors and local state estimators is vulnerable to DoS attacks. For resisting the impacts of DoS attacks, a compensation strategy is adopted. A new method has been developed to devise the local state estimators. The innovation signals from local estimators will be quantized first before entering the network. Gains of the local state estimators and the fusion weighting matrices are acquired by solving a linear matrix inequality. Finally, the effectiveness of the proposed methods is verified by a target tracking system.**

***Index Terms*—Denial-of-service (DoS) attacks, fusion estimation, networked systems, quantization, round-robin scheduling protocol (RRSP).**

Nomenclature

DoS	Denial of service.
RRSP	Round-robin scheduling protocol.
MSFE	Multisensor fusion estimation.
CPS	Cyber-physical system.
LSE	Local state estimate.
DFE	Distributed fusion estimate.
LMI	Linear matrix inequality.
MSE	Mean square error.

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I. Introduction

Research enthusiasm for MSFE has grown rapidly over the past few decades. The purpose of MSFE is to best utilize the data gathered by the distributed sensors and improve the estimation accuracy and reliability. Compared with the traditional single-sensor estimation, MSFE has higher accuracy due to its rich sources of information, comprehensive target perception, and strong fault-tolerant capability [1]. In recent years, various methods of fusion estimation for multisensor systems have emerged in an endless stream and have been applied in many fields, such as system monitoring [2], [3]; target localization [4]; signal processing [5]; fault detection [6], [7]; CPSs [8]; and multiagent systems [9], [10], [11].

The existing methods about MSFE can be divided into two categories: 1) centralized fusion estimation and 2) distributed fusion estimation. In the centralized structure, the measurements from all of the sensors are directly transmitted to the fusion center and then extended to high-dimensional measurements for further processing [12]. In contrast, in the distributed structure, each sensor sends its LSE to the fusion center, and then the fusion estimation is performed according to the specific fusion rules. Compared with the centralized fusion estimation, the distributed fusion estimation is not optimal, but its parallel structure makes it robust and flexible [13]. Therefore, many scholars and scientists have committed themselves to the research of the distributed fusion estimation and made a lot of achievements. For instance, for Gaussian white noise with known covariance matrix, a class of distributed fusion estimation algorithms based on the Kalman filter has been proposed in [8], [14], and [15]. For energy-bounded noise, many H_∞ fusion algorithms have been proposed in [3], [5], and [16]. There is still a kind of noise widely used in practical applications, which is bounded at any time, but the bound is unknown. Presently, there is not much work on the distributed fusion estimation for systems with bounded noise. How to design a distributed fusion estimation algorithm for systems with bounded noise is difficult and needs further exploration.

In MSFE systems, sensors, local estimators, and the fusion estimator are connected through a shared network, which has the advantages of convenience for remote operation, simple installation and maintenance, low cost, and resource sharing [12]. However, in the process of communication, the limited network bandwidth may lead to packet dropouts, disorder, and transmission delay [17], [18], [19], [20], which will degrade the performance of fusion estimation. In addition, the number and value of the state variables of networked systems may be large, so it is unrealistic to transmit each signal entirely to the other end of the network over a communication channel with limited bandwidth. As such, in MSFE systems, it makes practical sense to introduce quantization and communication protocols in response to the finite communication bandwidth. The essence of quantization is to divide the range of continuous variation of input signal amplitude into finite nonoverlapping subintervals. Each subinterval is represented by a certain value in the interval, and the input signal falling into it will be output with this value. Some results related to quantization have been presented in [21], [22], [23], and [24]. For example, in [22], the multibit decentralized detection is tackled for a noise-corrupted unknown signal parameter in sensor networks, where dumb sensor measurements are quantized before being sent to a fusion center. Innovation sequences are quantized by logarithmic quantizers to design local state estimators to overcome the unboundedness of unstable systems in [24]. Different from quantization, communication protocols are capable of handling the limited network bandwidth by coordinating the transmission sequence of the measurements of different sensors, among which RRSP is a widely implemented one to regulate the transmissions. Under the RRSP, each sensor node is assigned equal access to the communication network in terms of a predetermined periodic order. To date, a wealthy body of work has been done on the RRSP in [25], [26], [27], and [28]. However, the existing works either only focus on the distributed fusion estimation under quantization or only study the distributed fusion estimation under a certain communication protocol, and few papers unify quantization, communication protocol, and fusion estimation under a certain framework model, which is still a challenge.

In addition, the insertion of the network may make the MSFE systems exposed to malicious attacks, such as DoS attacks [8], [29], [30], [31]; deception attacks [32], [33], [34]; and replay attacks [35]. Among them, DoS attacks are the most harmful. The purpose of DoS attacks is to interfere with the data transmission among system components, so that the measurements and control signals cannot reach the devices at the other end. Over the past few decades, there has been great research progress on DoS attacks. For example, a security control approach has been proposed in [36] to defend against DoS attacks by fully utilizing the nonattack intervals. Moreover, the risk-sensitive stochastic control problem under DoS attacks has been considered in [37], where the enemies randomly held back the data packets by a hidden Markov model. Furthermore, a distributed framework has been developed in [38] so as to study the coordination behavior of multiagent systems when opponents initiated distributed DoS attacks. Nevertheless, with the consideration of the effects of RRSP and DoS attacks, the distributed fusion estimation for MSFE systems becomes more complex, which has not been adequately addressed and motivates our current research.

Based on the aforementioned analysis, this article will pay attention to the secure local state estimation and distributed fusion estimation for MSFE systems with bandwidth-constrained networks and DoS attacks. The main contributions of this article can be summarized as follows.

1) The RRSP and the quantization scheme are applied to decrease the negative impacts induced by the bandwidth-constrained networks. Different from some existing works, the signals to be quantized in this article are innovation signals, that is, the differences between measurements and the estimates of measurements.
2) A new method for designing local state estimators is proposed to guarantee the stability of dynamics of the discussed estimation error system.
3) A compensation strategy is employed for the sake of lessening the performance degradation caused by DoS attacks. Once the signals of sensors are blocked by DoS attacks, the historical data stored in the buffer will be utilized for compensation.
4) A novel distributed fusion estimation approach is presented for multisensor systems with unknown bounded noises under limited network bandwidth and DoS attacks, where the gains of the local state estimators and fusion weighting matrices are obtained by seeking the solution of an LMI. Compared to some existing results on the distributed fusion estimation, such as the classical distributed Kalman weighted fusion method in [8], [14], and [15] and some H_∞ fusion estimation algorithms in [3], [5], and [16], it reduces the requirement of system noise and has a wider application range.

The rest of this article is organized as follows. A distributed fusion estimation model based on the RRSP and the quantization scheme under DoS attacks is established in Section II. In Section III, the stable local state estimators and the distributed fusion estimator are designed. In Section IV, a target tracking system is used to verify the effectiveness of the proposed methods. Finally, Section V concludes this article.

Notations: $\mathbb{R}^n$ stands for the n-dimensional Euclidean space. T denotes the transpose of matrix, and $\mathbb{E}\{\cdot\}$ stands for expectation. I represents the identity matrix with appropriate dimension. The function mod (a_1, a_2) indicates the nonnegative remainder on division for a_1 by a_2. $\delta(\cdot)$ is the Kronecker delta function, and diag$\{\cdot\}$ means a block diagonal matrix. Prob$\{\cdot\}$ indicates the probability of the event. The symmetric terms in a symmetric matrix are denoted by $*$, and col$\{a_1, \ldots, a_n\}$ means a column vector, the elements of which are $a_1, \ldots, a_n$.

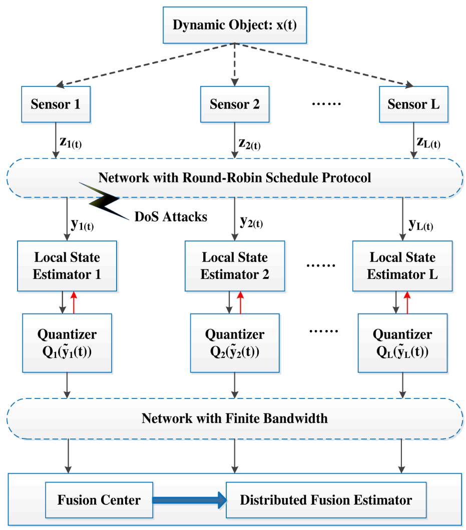

Fig. 1. Distributed fusion estimation structure.

II. Problem Formulation

Consider a time-varying target plant described by the following discrete linear model:

$$x(t+1) = A(t)x(t) + B(t)w(t) \tag{1}$$

$$z_i(t) = C_i(t)x(t) + v_i(t), \quad i = 1, 2, \ldots, L \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state of system at time t, and $z_i(t) \in \mathbb{R}^m$ is the measurement output of the ith sensor. L is the total number of sensors. $A(t)$, $B(t)$, and $C_i(t)$ are time-varying matrices with appropriate dimensions. $w(t) \in \mathbb{R}^l$ and $v_i(t) \in \mathbb{R}^m$ are bounded noises satisfying

$$w(t)^T w(t) \le \theta_w, v_i(t)^T v_i(t) \le \theta_{v_i} \tag{3}$$

where θ_w and θ_{v_i} are unknown scalars.

Assumption 1 (See [39]): The pairs $(A(t), C_i(t))$ are observable.

Remark 1: The bounded noise assumption comes from [40]. Such an assumption is mild. Bounded noise exists in intelligent vehicle localization systems, target tracking systems, and mobile robot experiments. Moreover, some current works have also studied this kind of noise [24], [41].

As shown in Fig. 1, the measurements of sensors are sent to a group of remote estimators through a shared network with limited bandwidth. In order to avoid data congestion and reduce the occurrence of data collisions, the RRSP is introduced to regulate the data communication in the channel connecting sensors and local state estimators. Under the RRSP, only one sensor's measurement signal is transmitted in the network at each moment. Define $h_t \in \{1, 2, \ldots, L\}$ as the sensor acquiring the network access at instant t. In other words, h_t determines which sensor is authorized to release packets. Obviously, it is satisfied that $h_t = h_{t+L}$. h_t can be expressed as follows:

$$h_t = \mathrm{mod}(t-1, L) + 1. \tag{4}$$

In fact, after being scheduled by the RRSP, the signal entering the network is $\delta(h_t - i)z_i(t)$ $(i = 1, 2, \ldots, L)$.

Remark 2: $\delta(h_t - i)$ indicates whether the measurement output of sensor i, i.e., $z_i(t)$, can enter the network for transmission at time t. If $\delta(h_t - i) = 1$, $z_i(t)$ will enter the network for transmission at time t. If $\delta(h_t - i) = 0$, $z_i(t)$ is not given access to the network, and the data stored in the buffer will be utilized.

When the measurements are delivered in the network, we need to take the security problems brought by the network into account. Adversaries may launch DoS attacks on the communication channel between sensors and local state estimators, resulting in the estimators unable to receive the data in time. Aiming to reflect the effect of DoS attacks on the system, the variable r_t is used to indicate whether DoS attacks occur or not, the value of which is either 0 or 1 [42]. Moreover, in response to the potential DoS attacks, received data at time $t-1$ will be used for compensation once the channel is attacked at time t. Let $y_i(t)$ denote the signal received by the ith local state estimator, and it can be modeled by

$$\begin{aligned} y_i(t) = r_t[&\delta(h_t - i)z_i(t) + (1 - \delta(h_t - i)) \\ &\times y_i(t-1)] + (1 - r_t)y_i(t-1) \end{aligned} \tag{5}$$

where r_t is a Bernoulli distributed random variable with Prob$\{r_t = 1\} = \alpha$ and Prob$\{r_t = 0\} = 1 - \alpha$ $(0 \le \alpha < 1)$.

Remark 3: Notice that (5) describes the situation that the transmission channel is under DoS attacks, which are governed by random variables. When $r_t = 0$, $y_i(t) = y_i(t-1)$, which implies that the adversaries have launched DoS attacks, making the communication data blocked. The measurements at instant $t-1$ are used as compensation. When $r_t = 1$, $y_i(t) = \delta(h_t - i)z_i(t)$ $+$ $(1 - \delta(h_t - i))y_i(t-1)$, which means that the estimators can normally receive the data conveyed via the network.

Define

$$X_i(t) = \begin{bmatrix} x(t) \\ y_i(t-1) \end{bmatrix}, \quad W_i(t) = \begin{bmatrix} w(t) \\ v_i(t) \end{bmatrix}.$$

The system model is rewritten as

$$\begin{aligned} X_i(t+1) = &\mathscr{A}_i(t)X_i(t) + \mathscr{B}_i(t)W_i(t) \\ &+ (r_t - \alpha)\mathscr{C}_i(t)X_i(t) \\ &+ (r_t - \alpha)\mathscr{D}_i(t)W_i(t) \end{aligned} \tag{6}$$

$$\begin{aligned} y_i(t) = &\mathscr{E}_i(t)X_i(t) + \alpha\delta(h_t - i)v_i(t) \\ &+ (r_t - \alpha)\mathscr{F}_i(t)X_i(t) \\ &+ (r_t - \alpha)\delta(h_t - i)v_i(t) \end{aligned} \tag{7}$$

where

$$\mathscr{A}_i(t) = \begin{bmatrix} A(t) & 0 \\ \alpha\delta(h_t - i)C_i(t) & (1 - \alpha\delta(h_t - i))I \end{bmatrix}$$

$$\mathscr{B}_i(t) = \begin{bmatrix} B(t) & 0 \\ 0 & \alpha\delta(h_t - i)I \end{bmatrix}$$

$$\mathscr{C}_i(t) = \begin{bmatrix} 0 & 0 \\ \delta(h_t - i)C_i(t) & -\delta(h_t - i))I \end{bmatrix}$$

$$\mathscr{D}_i(t) = \begin{bmatrix} 0 & 0 \\ 0 & \delta(h_t - i))I \end{bmatrix}$$

$$\mathscr{E}_i(t) = \begin{bmatrix} \alpha\delta(h_t - i)C_i(t) & (1 - \alpha\delta(h_t - i))I \end{bmatrix}$$

$$\mathscr{F}_i(t) = \begin{bmatrix} \delta(h_t - i)C_i(t) & -\delta(h_t - i))I \end{bmatrix}.$$

Due to the limited network bandwidth, the signals will be quantized first and then transmitted to the fusion center. It is required that the signal to be quantized in this article is $\tilde{y}_i(t)$, which is defined by $\tilde{y}_i(t) = y_i(t) - \alpha\delta(h_t - i)C_i(t)\hat{x}_i(t) - (1 - \alpha\delta(h_t - i))\hat{y}_i(t-1)$, where $\hat{x}_i(t)$ is the estimate of $x(t)$ by the ith local estimator and $\hat{y}_i(t)$ represents the estimate of $y_i(t)$. The logarithmic quantization scheme, modeled as $Q_i(\cdot) = \mathrm{col}\,\{q_{i1}(\cdot), q_{i2}(\cdot), \ldots, q_{im}(\cdot)\} \in \mathbb{R}^m$, is aimed at quantizing the signal $\tilde{y}_i(t)$, and $q_{ij}(\cdot) \in \mathbb{R}$, $j \in \{1, 2, \ldots, m\}$, is designed for quantizing the jth part of the signal $\tilde{y}_i(t)$.

Remark 4: The common quantization strategies are broadly divided into two categories: one is uniform quantization and the other is logarithmic quantization. When the input is close to the origin, the signal-to-noise ratio of uniform quantization is very small, which will have an unfavorable effect on the estimation performance. Compared with the uniform quantization, logarithmic quantization has better signal-to-noise ratio for smaller inputs. Consequently, this article adopts the logarithmic quantization strategy for design.

Define the set of the quantization level of $q_{ij}(\cdot)$ as follows:

$$U_{ij} = \{\pm u_h^{(ij)} : u_h^{(ij)} = \rho_{ij}^h u_0^{(ij)}, h = 0, \pm 1, \pm 2, \ldots\} \cup \{0\}$$
$$(0 < \rho_{ij} < 1, u_0^{(ij)} > 0)$$

where ρ_{ij} is the density of quantization. Then, $q_{ij}(\cdot)$ is designed as

$$q_{ij}(\epsilon) = \begin{cases} u_h^{(ij)}, & \text{if } \frac{1}{1+\xi_{ij}} u_h^{(ij)} < \epsilon \le \frac{1}{1-\xi_{ij}} u_h^{(ij)} \\ 0, & \text{if } \epsilon = 0 \\ -q_{ij}(-\epsilon), & \text{if } \epsilon < 0 \end{cases} \tag{8}$$

where $\xi_{ij} = [(1 - \rho_{ij})/(1 + \rho_{ij})](0 < \xi_{ij} < 1)$. Similar to [43], $q_{ij}(\epsilon)$ can be rewritten as $q_{ij}(\epsilon) = (1 + \Gamma_{ij}(t))\epsilon$ for certain $\Gamma_{ij}(t)$, which satisfies $|\Gamma_{ij}(t)| \le \xi_{ij}$. It is not difficult to find that smaller ρ_{ij} or larger ξ_{ij} will bring about rough quantization, and the length of the transmitted packet will decrease with the increase of ξ_{ij}, which is prespecified in this article.

Based on (8), $Q_i[\tilde{y}_i(t)]$ can be expressed as

$$Q_i[\tilde{y}_i(t)] = (I + \Gamma_i(t))\tilde{y}_i(t) \tag{9}$$

where

$$\Gamma_i(t) = \mathrm{diag}\{\Gamma_{i1}(t), \Gamma_{i2}(t), \ldots, \Gamma_{im}(t)\}.$$

After $\tilde{y}_i(t)$ is quantized and transmitted to the fusion center through the network, the estimates of $x(t)$ and $y_i(t-1)$ in fusion center, expressed as $\hat{x}_{fi}(t)$ and $\hat{y}_{fi}(t-1)$, are designed as

$$\hat{x}_{fi}(t+1) = A(t)\hat{x}_{fi}(t) + K_i^x(t+1)Q_i[\tilde{y}_i(t)] \tag{10}$$

$$\begin{aligned} \hat{y}_{fi}(t) = {} & \alpha\delta(h_t - i)C_i(t)\hat{x}_{fi}(t) \\ & + (1 - \alpha\delta(h_t - i))\hat{y}_{fi}(t-1) \\ & + K_i^y(t)Q_i[\tilde{y}_i(t)] \end{aligned} \tag{11}$$

where the local estimator gain matrices $K_i^x(t+1)$ and $K_i^y(t)$ are unknown and will be designed to minimize the upper bound of the estimation error.

Define

$$\hat{X}_{fi}(t) = \begin{bmatrix} \hat{x}_{fi}(t) \\ \hat{y}_{fi}(t-1) \end{bmatrix}, \quad K_i(t) = \begin{bmatrix} K_i^x(t) \\ K_i^y(t-1) \end{bmatrix}.$$

Then, (10) and (11) can be rewritten as

$$\hat{X}_{fi}(t+1) = \mathscr{A}_i(t)\hat{X}_{fi}(t) + K_i(t+1)Q_i[\tilde{y}_i(t)] \tag{12}$$

$$\hat{x}_{fi}(t) = [I \quad 0]\, \hat{X}_{fi}(t). \tag{13}$$

In order to get rid of the quantization impacts, the local estimator of each sensor is designed as follows:

$$\hat{x}_i(t+1) = A(t)\hat{x}_i(t) + K_i^x(t+1)Q_i[\tilde{y}_i(t)] \tag{14}$$

$$\begin{aligned} \hat{y}_i(t) = {} & \alpha\delta(h_t - i)C_i(t)\hat{x}_i(t) \\ & + (1 - \alpha\delta(h_t - i))\hat{y}_i(t-1) \\ & + K_i^y(t)Q_i[\tilde{y}_i(t)] \end{aligned} \tag{15}$$

where $\hat{x}_i(t)$ is the local estimate of $x(t)$ for sensor i and $\hat{y}_i(t)$ is the estimate of $y_i(t)$. Furthermore, it is required that the initial values of $\hat{x}_i(t)$ and $\hat{y}_i(t-1)$ are the same as the initial values of $\hat{x}_{fi}(t)$ and $\hat{y}_{fi}(t-1)$, respectively.

Define $\hat{X}_i(t) = \begin{bmatrix} \hat{x}_i^T(t) & \hat{y}_i^T(t-1) \end{bmatrix}^T$. Then, (14) and (15) can be rewritten as

$$\hat{X}_i(t+1) = \mathscr{A}_i(t)\hat{X}_i(t) + K_i(t+1)Q_i[\tilde{y}_i(t)] \tag{16}$$

$$\hat{x}_i(t) = [I \quad 0]\, \hat{X}_i(t). \tag{17}$$

Remark 5: Most previous studies use the local state estimator designed as $\hat{X}_i(t+1) = \mathscr{A}_i(t)\hat{X}_i(t) + K_i(t+1)\tilde{y}_i(t)$. However, the stability of $\hat{X}_{fi}(t)$ at the fusion center cannot be guaranteed due to the involvement of state-related noises. In view of this situation, (14) and (15) are presented.

Based on $\hat{x}_{fi}(t)$ in the fusion center, the DFE of $x(t)$ is expressed as

$$\hat{x}(t) = \sum_{i=1}^{L} \Omega_i(t)\hat{x}_{fi}(t) \tag{18}$$

where the sum of the fusion weighting matrix $\Omega_i(t)$ $(i = 1, 2, \ldots, L)$ is I, and $\Omega_i(t)$ will be devised in the following section.

Remark 6: Although some distributed fusion estimation problems have been conducted in [41], [44], and [45], the addressed issue in this article is different from the existing ones. In [41], the distributed fusion estimation for nonlinear systems with unknown noise statistics was investigated. In [44], the distributed robust fusion estimation for multisensor systems with parameter uncertainties was studied. In [45], the distributed fusion estimation for multisensor multirate systems with correlated noises was researched. However, the aforementioned references are based on the assumption that the network-based communication resources are not limited, and the addressed systems work in safe environments, which is actually unrealistic. Therefore, in order to eliminate the impact of precious network resources and DoS attacks, a new distributed security fusion estimation method is proposed in this article for networked multisensor systems.

Define $e_i(t) = x(t) - \hat{x}_i(t)$ as the local estimation error and $E_i(t) = X_i(t) - \hat{X}_i(t)$ as the augmented local estimation error. Substituting (6), (7), (9), and (16) into the definition of $E_i(t)$ yields

$$\begin{aligned} E_i(t+1) &= M_i(t+1)E_i(t) + G_i(t+1)W_i(t) \\ &\quad + (r_t - \alpha)N_i(t+1)X_i(t) \\ &\quad + (r_t - \alpha)H_i(t+1)W_i(t) \end{aligned} \tag{19}$$

where

$$\begin{aligned} M_i(t) &= \mathscr{A}_i(t-1) - K_i(t)(I + \Gamma_i(t))\mathscr{E}_i(t-1) \\ N_i(t) &= \mathscr{C}_i(t-1) - K_i(t)(I + \Gamma_i(t))\mathscr{F}_i(t-1) \\ G_i(t) &= \mathscr{B}_i(t-1) - \alpha\delta(h_{t-1} - i)K_i(t)(I + \Gamma_i(t))\vec{I} \\ H_i(t) &= \mathscr{D}_i(t-1) - \delta(h_{t-1} - i)K_i(t)(I + \Gamma_i(t))\vec{I} \\ \vec{I} &= [0 \quad I]. \end{aligned}$$

Let $\hat{E}_i(t) = \begin{bmatrix} E_i^T(t) & X_i^T(t) \end{bmatrix}^T$. Then, in terms of (6) and (19), it can be derived that

$$\begin{aligned} \hat{E}_i(t+1) &= M_i^{\mathscr{A}}(t+1)\hat{E}_i(t) + G_i^{\mathscr{B}}(t+1)W_i(t) \\ &\quad + (r_t - \alpha)N_i^{\mathscr{C}}(t+1)\hat{E}_i(t) \\ &\quad + (r_t - \alpha)H_i^{\mathscr{D}}(t+1)W_i(t) \end{aligned} \tag{20}$$

where

$$M_i^{\mathscr{A}}(t) = \begin{bmatrix} M_i(t) & 0 \\ 0 & \mathscr{A}_i(t-1) \end{bmatrix}, \quad G_i^{\mathscr{B}}(t) = \begin{bmatrix} G_i(t) \\ \mathscr{B}_i(t-1) \end{bmatrix}$$

$$N_i^{\mathscr{C}}(t) = \begin{bmatrix} 0 & N_i(t) \\ 0 & \mathscr{C}_i(t-1) \end{bmatrix}, \quad H_i^{\mathscr{D}}(t) = \begin{bmatrix} H_i(t) \\ \mathscr{D}_i(t-1) \end{bmatrix}.$$

Define the fusion estimation error $e(t) = x(t) - \hat{x}(t)$. From the definitions of $\hat{E}_i(t)$, it is easy to get

$$e(t) = \sum_{i=1}^{L} \Omega_i(t)\hat{I}\hat{E}_i(t) \tag{21}$$

where $\hat{I} = [I \quad 0]$.

Then, combining (20) and (21), the fusion error system is constructed as

$$\begin{aligned} \hat{E}_F(t+1) &= [\widetilde{M}(t+1) + (r_t - \alpha)\widetilde{N}(t+1)]\hat{E}_F(t) \\ &\quad + [\widetilde{G}(t+1) + (r_t - \alpha)\widetilde{H}(t+1)]W_F(t) \end{aligned} \tag{22}$$

$$e(t) = \Omega(t)\hat{E}_F(t) \tag{23}$$

where

$$\hat{E}_F(t) = \begin{bmatrix} \hat{E}_1(t) \\ \vdots \\ \hat{E}_L(t) \end{bmatrix}, \quad W_F(t) = \begin{bmatrix} W_1(t) \\ \vdots \\ W_L(t) \end{bmatrix}$$

$$\begin{aligned} \widetilde{M}(t) &= \text{diag}\{M_1^{\mathscr{A}}(t), \ldots, M_L^{\mathscr{A}}(t)\} \\ \widetilde{N}(t) &= \text{diag}\{N_1^{\mathscr{C}}(t), \ldots, N_L^{\mathscr{C}}(t)\} \\ \widetilde{G}(t) &= \text{diag}\{G_1^{\mathscr{B}}(t), \ldots, G_L^{\mathscr{B}}(t)\} \\ \widetilde{H}(t) &= \text{diag}\{H_1^{\mathscr{D}}(t), \ldots, H_L^{\mathscr{D}}(t)\} \\ \Omega(t) &= \left[\Omega_1(t)\hat{I}, \ldots, (I - \textstyle\sum_{i=1}^{L-1} \Omega_i(t))\hat{I}\right]. \end{aligned}$$

The main objectives of this article are as follows.

1) Find out the local estimator gain $K_i(t)$ and the distributed fusion weighting matrix $\Omega_i(t)$ such that the fusion error system (23) is asymptotically stable.
2) Under the zero-initial condition, the fusion error system satisfies

$$\sum_{t=0}^{\infty} \mathbb{E}\{e^T(t)e(t)\} < \eta^2 \sum_{t=0}^{\infty} \mathbb{E}\{W_F^T(t)W_F(t)\} \tag{24}$$

where η is a predetermined H_∞ performance level.

III. Main Results

Before proceeding further, the following lemma needs to be given, which will be used in the subsequent sections.

Lemma 1 (See [17]): Suppose that Υ_1, Υ_2, and Υ_3 are given matrices appropriately dimensioned and $\Upsilon_1 = \Upsilon_1^T$. Then

$$\Upsilon_1 + \Upsilon_3\Lambda(t)\Upsilon_2 + \Upsilon_2^T\Lambda(t)^T\Upsilon_3^T < 0$$

is true for $\Lambda(t)$, which satisfies $\Lambda(t)^T\Lambda(t) \le I$ if and only if there exists $\zeta > 0$ such that

$$\Upsilon_1 + \zeta^{-1}\Upsilon_3\Upsilon_3^T + \zeta\Upsilon_2^T\Upsilon_2 < 0.$$

Theorem 1: For the given attack probability $\alpha (0 \le \alpha < 1)$, quantization parameter $\xi_{ij} (i = 1, \ldots, L; j = 1, \ldots, m)$, local estimator gain $K_i(t+1)$, and H_∞ performance index η, if there exist a positive scalar τ, positive-definite matrices $\Xi_{h_t}^{i1}$, $\Xi_{h_t}^{i2}$ $(i = 1, \ldots, L)$, and matrices $\Omega_1(t), \ldots, \Omega_{L-1}(t)$ and $I - \sum_{i=1}^{L-1} \Omega_i(t)$ with appropriate dimensions such that

$$\begin{bmatrix} -\tau I & * & * \\ \tau\Delta_2^T(t+1) & \Delta_1(t+1) & * \\ 0 & \Delta_3^T(t+1) & -\tau I \end{bmatrix} < 0 \tag{25}$$

where

$$\Delta_1(t) = \begin{bmatrix} -\Xi_{h_{t-1}} & * & * & * & * \\ 0 & -\eta^2 I & * & * & * \\ D(t) & P(t) & -\Xi_{h_t} & * & * \\ \bar{\alpha}R(t) & \bar{\alpha}S(t) & 0 & -\Xi_{h_t} & * \\ \Omega(t-1) & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\Delta_2(t) = \tau^{-1}\begin{bmatrix} 0 & 0 & -\Phi^T(t) & 0 & 0 \\ 0 & 0 & 0 & -\bar{\alpha}\Phi^T(t) & 0 \end{bmatrix}$$

$$\Delta_3(t) = \begin{bmatrix} \tau Z\vec{\mathscr{E}}(t-1) & \alpha\tau Z\Psi_{h_{t-1}} & 0 & 0 & 0 \\ \tau Z\vec{\mathscr{F}}(t-1) & \tau Z\Psi_{h_{t-1}} & 0 & 0 & 0 \end{bmatrix}^T$$

$$\Xi_{h_t} = \mathrm{diag}\{\Xi_{h_t}^1, \ldots, \Xi_{h_t}^L\}, \quad \Xi_{h_t}^i = \mathrm{diag}\{\Xi_{h_t}^{i1}, \Xi_{h_t}^{i2}\}$$

$$\bar{\alpha} = \sqrt{\alpha - \alpha^2}, \quad D(t) = \mathrm{diag}\{D_1(t), \ldots, D_L(t)\}$$

$$D_i(t) = \begin{bmatrix} D_i^1(t) & 0 \\ 0 & \Xi_{h_t}^{i2}\mathscr{A}_i(t-1) \end{bmatrix}$$

$$D_i^1(t) = \Xi_{h_t}^{i1}\mathscr{A}_i(t-1) - \Xi_{h_t}^{i1}K_i(t)\mathscr{E}_i(t-1)$$

$$P(t) = \mathrm{diag}\{P_1(t), \ldots, P_L(t)\}$$

$$P_i(t) = \begin{bmatrix} P_i^1(t) \\ \Xi_{h_t}^{i2}\mathscr{B}_i(t-1) \end{bmatrix}$$

$$P_i^1(t) = \Xi_{h_t}^{i1}\mathscr{B}_i(t-1) - \Xi_{h_t}^{i1}K_i(t)\alpha\delta(h_{t-1}-i)\vec{I}$$

$$R(t) = \mathrm{diag}\{R_1(t), \ldots, R_L(t)\}$$

$$R_i(t) = \begin{bmatrix} 0 & R_i^1(t) \\ 0 & \Xi_{h_t}^{i2}\mathscr{C}_i(t-1) \end{bmatrix}$$

$$R_i^1(t) = \Xi_{h_t}^{i1}\mathscr{C}_i(t-1) - \Xi_{h_t}^{i1}K_i(t)\mathscr{F}_i(t-1)$$

$$S(t) = \mathrm{diag}\{S_1(t), \ldots, S_L(t)\}$$

$$S_i(t) = \begin{bmatrix} S_i^1(t) \\ \Xi_{h_t}^{i2}\mathscr{D}_i(t-1) \end{bmatrix}$$

$$S_i^1(t) = \Xi_{h_t}^{i1}\mathscr{D}_i(t-1) - \Xi_{h_t}^{i1}K_i(t)\delta(h_{t-1}-i)\vec{I}$$

$$\Phi(t) = \mathrm{diag}\{\Phi_1(t), \ldots, \Phi_L(t)\}$$

$$\Phi_i(t) = \begin{bmatrix} \Xi_{h_t}^{i1}K_i(t) \\ 0 \end{bmatrix}$$

$$Z = \mathrm{diag}\{Z_1, \ldots, Z_L\}, \quad Z_i = \mathrm{diag}\{\xi_{i1}, \ldots, \xi_{im}\}$$

$$\vec{\mathscr{E}}(t) = \mathrm{diag}\{\vec{\mathscr{E}}_1(t), \ldots, \vec{\mathscr{E}}_L(t)\}, \quad \vec{\mathscr{E}}_i(t) = \begin{bmatrix} \mathscr{E}_i(t) & 0 \end{bmatrix}$$

$$\vec{\mathscr{F}}(t) = \mathrm{diag}\{\vec{\mathscr{F}}_1(t), \ldots, \vec{\mathscr{F}}_L(t)\}, \quad \vec{\mathscr{F}}_i(t) = \begin{bmatrix} 0 & \mathscr{F}_i(t) \end{bmatrix}$$

$$\Psi_{h_t} = \mathrm{diag}\{\delta(h_t-1)\vec{I}, \ldots, \delta(h_t-L)\vec{I}\}$$

then the fusion error system (23) is asymptotically stable under the H_∞ performance level η. Moreover, the distributed fusion weighting matrices are $\Omega_1(t), \ldots, \Omega_{L-1}(t), I - \sum_{i=1}^{L-1}\Omega_i(t)$.

Proof: See Appendix A.

Theorem 2: For the given attack probability $\alpha(0 \le \alpha < 1)$, quantization parameter $\xi_{ij}(i = 1, \ldots, L; j = 1, \ldots, m)$, and H_∞ performance index η, if there exist a positive scalar τ, positive-definite matrices $\Xi_{h_t}^{i1}$, $\Xi_{h_t}^{i2}$ $(i = 1, \ldots, L)$, and matrices $\Pi_i(t+1)$, $\Omega_1(t), \ldots, \Omega_{L-1}(t)$ and $I - \sum_{i=1}^{L-1}\Omega_i(t)$ with appropriate dimensions such that

$$\begin{bmatrix} -\tau I & * & * \\ \tau\widehat{\Delta}_2^T(t+1) & \widehat{\Delta}_1(t+1) & * \\ 0 & \Delta_3^T(t+1) & -\tau I \end{bmatrix} < 0 \tag{26}$$

where

$$\widehat{\Delta}_1(t) = \begin{bmatrix} -\Xi_{h_{t-1}} & * & * & * & * \\ 0 & -\eta^2 I & * & * & * \\ \hat{D}(t) & \hat{P}(t) & -\Xi_{h_t} & * & * \\ \bar{\alpha}\hat{R}(t) & \bar{\alpha}\hat{S}(t) & 0 & -\Xi_{h_t} & * \\ \Omega(t-1) & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\widehat{\Delta}_2(t) = \tau^{-1}\begin{bmatrix} 0 & 0 & -\hat{\Phi}^T(t) & 0 & 0 \\ 0 & 0 & 0 & -\bar{\alpha}\hat{\Phi}^T(t) & 0 \end{bmatrix}$$

$$\hat{D}(t) = \mathrm{diag}\{\hat{D}_1(t), \ldots, \hat{D}_L(t)\}$$

$$\hat{D}_i(t) = \begin{bmatrix} \Xi_{h_t}^{i1}\mathscr{A}_i(t-1) - \Pi_i(t)\mathscr{E}_i(t-1) & 0 \\ 0 & \Xi_{h_t}^{i2}\mathscr{A}_i(t-1) \end{bmatrix}$$

$$\hat{P}(t) = \mathrm{diag}\{\hat{P}_1(t), \ldots, \hat{P}_L(t)\}$$

$$\hat{P}_i(t) = \begin{bmatrix} \Xi_{h_t}^{i1}\mathscr{B}_i(t-1) - \Pi_i(t)\alpha\delta(h_{t-1}-i)\vec{I} \\ \Xi_{h_t}^{i2}\mathscr{B}_i(t-1) \end{bmatrix}$$

$$\hat{R}(t) = \mathrm{diag}\{\hat{R}_1(t), \ldots, \hat{R}_L(t)\}$$

$$\hat{R}_i(t) = \begin{bmatrix} 0 & \Xi_{h_t}^{i1}\mathscr{C}_i(t-1) - \Pi_i(t)\mathscr{F}_i(t-1) \\ 0 & \Xi_{h_t}^{i2}\mathscr{C}_i(t-1) \end{bmatrix}$$

$$\hat{S}(t) = \mathrm{diag}\{\hat{S}_1(t), \ldots, \hat{S}_L(t)\}$$

$$\hat{S}_i(t) = \begin{bmatrix} \Xi_{h_t}^{i1}\mathscr{D}_i(t-1) - \Pi_i(t)\delta(h_{t-1}-i)\vec{I} \\ \Xi_{h_t}^{i2}\mathscr{D}_i(t-1) \end{bmatrix}$$

$$\hat{\Phi}(t) = \mathrm{diag}\{\hat{\Phi}_1(t), \ldots, \hat{\Phi}_L(t)\}$$

$$\hat{\Phi}_i(t) = \begin{bmatrix} \Pi_i(t) \\ 0 \end{bmatrix}$$

then the fusion error system (23) is asymptotically stable under the H_∞ performance level η. In this case, the local estimator gain matrices can be obtained by

$$K_i(t) = (\Xi_{h_t}^{i1})^{-1}\Pi_i(t). \tag{27}$$

Proof: See Appendix B.

IV. Simulation Examples

In this section, a linear target tracking system composed of two sensors is considered.

Denote the state $x(t) = \mathrm{col}\,\{x^1(t), x^2(t), x^3(t), x^4(t)\}$, and all the parameters used in the simulation are given in Table I. Set the initial values $x(0) = \mathrm{col}\,\{0.7, -0.7, 0.7, -0.7\}$, $\hat{x}_1(0) = \hat{x}_{f1}(0) = \mathrm{col}\ \{0.85, -0.85, 0.85, -0.85\}$, and $\hat{x}_2(0) = \hat{x}_{f2}(0) = \mathrm{col}\{0.45, -0.45, 0.45, -0.45\}$. Since $w(t)$ and $v_i(t)$

TABLE I
PARAMETER NAMES AND CORRESPONDING VALUES

Parameter name	Parameter value
$A(t)$	$\begin{bmatrix} 0.82 & 0.01 & 0 & 0.04 \\ 0.03 & -0.79+0.12\sin(t) & 0.17 & 0 \\ 0.11 & 0 & 0.63 & 0.05 \\ 0 & 0.07 & 0.01 & 0.85-0.12\sin(t) \end{bmatrix}$
$B(t)$	$\begin{bmatrix} 0.8+0.11\sin(t) \\ 0.7+0.18\cos(t) \\ 0.8-0.14\sin(t) \\ 0.7-0.17\cos(t) \end{bmatrix}$
$C_1(t)$	$\begin{bmatrix} 0.8+0.1\sin(t) & 0.57 & 0.12 & 0.3 \\ 0.09 & 0.23 & 0.7 & 0.64 \end{bmatrix}$
$C_2(t)$	$\begin{bmatrix} 0.6-0.1\cos(t) & 0.15 & 0.11 & 0.29 \\ 0.21 & 0.83 & 0.65 & 0.7 \end{bmatrix}$
ρ_{11}	0.5
ρ_{12}	0.5
ρ_{21}	0.5
ρ_{22}	0.5
$u_0^{(11)}$	0.2
$u_0^{(12)}$	0.2
$u_0^{(21)}$	0.2
$u_0^{(22)}$	0.2
α	0.8
ξ_{11}	0.5
ξ_{12}	0.5
ξ_{21}	0.6
ξ_{22}	0.6
η	0.9999

are bounded noises, the values of them can be generated by the function “rand” of MATLAB.[1]

By means of the LMI toolbox of MATLAB, the distributed fusion weighting matrix $\Omega_i(t)$ can be obtained, some of which are listed in Table II.

In total, 50 tests have been conducted in order to avoid one-time occasionality. The trajectories in each dimension of the system state $x(t)$ and the corresponding DFE $\hat{x}(t)$ subject to DoS attacks and RRSP are, respectively, depicted in Figs. 2–5. It can be observed that no matter in which dimension the fusion estimation errors are very small and hardly exceed 0.2.

In order to better reflect the estimation performance of the two local state estimators and the fusion estimator, an MSE as an evaluation index is introduced. The MSE for LSE is defined by

$$\vec{e}_i(t) = \mathbb{E}\{(x(t)-\hat{x}_i(t))^T (x(t)-\hat{x}_i(t))\}. \tag{28}$$

Similarly, the MSE for DFE is given by

$$\vec{e}(t) = \mathbb{E}\{(x(t)-\hat{x}(t))^T (x(t)-\hat{x}(t))\}. \tag{29}$$

MSEs for the two local estimates and DFE are shown in Fig. 6. It is not difficult to see that the MSEs for the two local estimated values and DFE are consistently lower than 0.07. When $t \in [0, 10]$, there is a significant difference in the MSE between the two local estimates, and the MSE for DFE is smaller than that for any LSE, which verifies the validity of the distributed fusion estimation method put forward in this article. When $t \in [11, 50]$, the MSEs of both the local estimates are very small, and they are basically consistent at each moment, so the MSE of DFE is roughly the same as both. In fact, the MSE for DFE may sometimes be slightly larger than that for a certain LSE. This is because the distributed fusion estimation is usually not optimal,

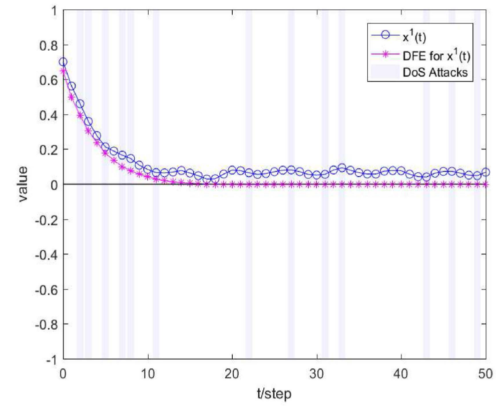

Fig. 2. Trajectories of the state $x^1(t)$ and DFE for $x^1(t)$.

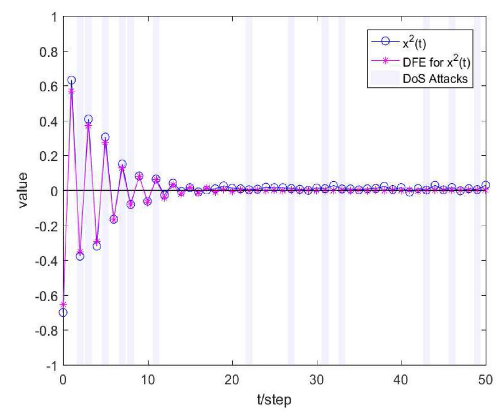

Fig. 3. Trajectories of the state $x^2(t)$ and DFE for $x^2(t)$.

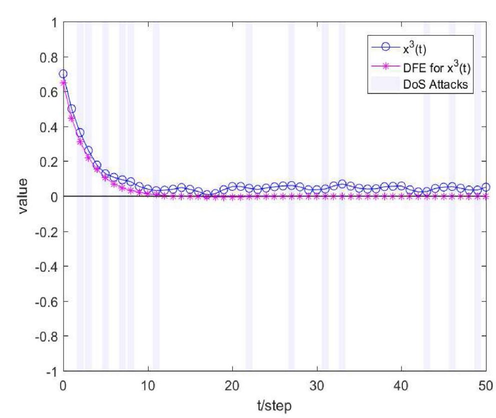

Fig. 4. Trajectories of the state $x^3(t)$ and DFE for $x^3(t)$.

[1]Considering the role of “rand” function, the bounded noises in the system are set as $w(t) = 0.02(\mathrm{rand}() + \sin(t))$, $v_1(t) = \mathrm{col}\{0.025(\mathrm{rand}() + \sin(t)), 0.025(\mathrm{rand}() + \sin(t))\}$ and $v_2(t) = \mathrm{col}\{-0.025(\mathrm{rand}() + \sin(t)), -0.025(\mathrm{rand}() + \sin(t))\}$.

TABLE II
FUSION WEIGHTING MATRICES

t	$\Omega_1(t)$	$\Omega_2(t)$
t=0	$\begin{bmatrix} 0.4914 & -0.0007 & -0.0067 & -0.0075 \\ -0.0128 & 0.4944 & -0.0179 & -0.0167 \\ -0.0059 & -0.0034 & 0.4907 & -0.0085 \\ -0.0101 & -0.0044 & -0.0139 & 0.4869 \end{bmatrix}$	$\begin{bmatrix} 0.5086 & 0.0007 & 0.0067 & 0.0075 \\ 0.0128 & 0.5056 & 0.0179 & 0.0167 \\ 0.0059 & 0.0034 & 0.5093 & 0.0085 \\ 0.0101 & 0.0044 & 0.0139 & 0.5131 \end{bmatrix}$
t=1	$\begin{bmatrix} 0.4948 & -0.0059 & -0.0003 & -0.0016 \\ -0.0055 & 0.4923 & -0.0033 & -0.0041 \\ -0.0002 & -0.0026 & 0.4967 & -0.0028 \\ -0.0021 & -0.0049 & -0.0040 & 0.4961 \end{bmatrix}$	$\begin{bmatrix} 0.5052 & 0.0059 & 0.0003 & 0.0016 \\ 0.0055 & 0.5077 & 0.0033 & 0.0041 \\ 0.0002 & 0.0026 & 0.5033 & 0.0028 \\ 0.0021 & 0.0049 & 0.0040 & 0.5039 \end{bmatrix}$
$\vdots$	$\vdots$	$\vdots$
t=49	$\begin{bmatrix} 0.4947 & -0.0057 & -0.0002 & -0.0017 \\ -0.0048 & 0.4936 & -0.0025 & -0.0036 \\ 0.0001 & -0.0020 & 0.4971 & -0.0026 \\ -0.0021 & -0.0046 & -0.0039 & 0.4959 \end{bmatrix}$	$\begin{bmatrix} 0.5053 & 0.0057 & 0.0002 & 0.0017 \\ 0.0048 & 0.5064 & 0.0025 & 0.0036 \\ -0.0001 & 0.0020 & 0.5029 & 0.0026 \\ 0.0021 & 0.0046 & 0.0039 & 0.5041 \end{bmatrix}$
t=50	$\begin{bmatrix} 0.4920 & -0.0008 & -0.0064 & -0.0069 \\ -0.0121 & 0.4941 & -0.0174 & -0.0158 \\ -0.0057 & -0.0036 & 0.4907 & -0.0082 \\ -0.0098 & -0.0047 & -0.0139 & 0.4874 \end{bmatrix}$	$\begin{bmatrix} 0.5080 & 0.0008 & 0.0064 & 0.0069 \\ 0.0121 & 0.5059 & 0.0174 & 0.0158 \\ 0.0057 & 0.0036 & 0.5093 & 0.0082 \\ 0.0098 & 0.0047 & 0.0139 & 0.5126 \end{bmatrix}$

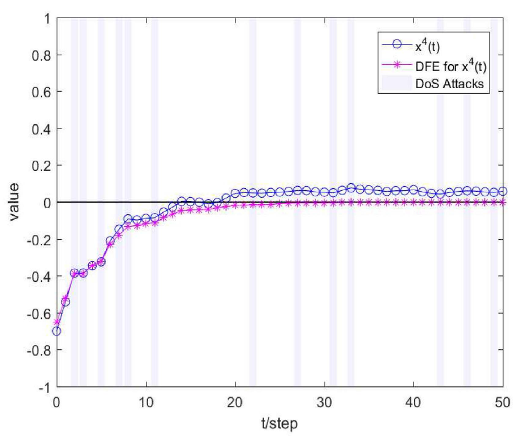

Fig. 5. Trajectories of the state $x^4(t)$ and DFE for $x^4(t)$.

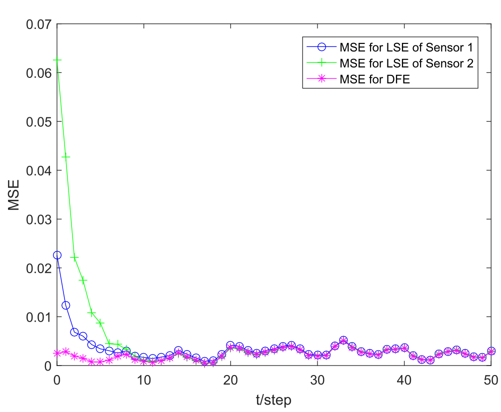

Fig. 6. MSE for LSE of Sensor 1, LSE of Sensor 2, and DFE.

but its structure makes it flexible and robust, which is one of the reasons why the distributed fusion estimation has been popular in recent years.

V. CONCLUSION

In this article, a distributed fusion estimation design approach for networked multisenor systems with bandwidth constraints has been proposed. The RRSP and the quantization scheme have been introduced to alleviate the communication pressure. Aiming at mitigating the impacts of DoS attacks on the estimation performance, a compensation strategy has been adopted. A new method has been developed to devise the local state estimators so that the stability of the discussed fusion error system can be guaranteed. Based on a certain LMI, the local estimator gains and fusion weighting matrices have been obtained. Eventually, an object tracking system has been used to prove the effectiveness of the proposed method.

What is noteworthy is that different types of attacks may occur in sensor networks, affecting the security of sensor networks. Therefore, the secure MSFE for bandwidth-constrained sensor networks under hybrid attacks and the FlexRay protocol is one of our future works.

APPENDIX A
PROOF OF THEOREM 1

Construct the following Lyapunov function:

$$\mathscr{L}(t) = \hat{E}_F^T(t)\Xi_{h_t}\hat{E}_F(t). \tag{30}$$

The difference of $\mathscr{L}(t)$ can be computed by

$$\Delta\mathscr{L}(t) = \hat{E}_F^T(t+1)\Xi_{h_{t+1}}\hat{E}_F(t+1) - \hat{E}_F^T(t)\Xi_{h_t}\hat{E}_F(t). \tag{31}$$

Define $\mathbb{S}_\kappa = \sum_{t=0}^{\kappa} \mathbb{E}\{e^T(t)e(t) - \eta^2 W_F^T(t)W_F(t)\}$. Then, one has

$$\begin{aligned}\mathbb{S}_\kappa &= \sum_{t=0}^{\kappa} \mathbb{E}\{e^T(t)e(t) - \eta^2 W_F^T(t)W_F(t) + \Delta\mathscr{L}(t)\}\\ &\quad - \sum_{t=0}^{\kappa}\mathbb{E}\{\Delta\mathscr{L}(t)\}\\ &= \sum_{t=0}^{\kappa} \mathbb{E}\{e^T(t)e(t) - \eta^2 W_F^T(t)W_F(t) + \Delta\mathscr{L}(t)\}\\ &\quad - \mathbb{E}\{\mathscr{L}(\kappa+1) - \mathscr{L}(0)\}.\end{aligned} \tag{32}$$

Under the zero-initial condition

$$\begin{aligned}\mathbb{S}_\kappa &= \sum_{t=0}^{\kappa} \mathbb{E}\{e^T(t)e(t) - \eta^2 W_F^T(t)W_F(t) + \Delta\mathscr{L}(t)\}\\ &\quad - \mathbb{E}\{\mathscr{L}(\kappa+1)\}.\end{aligned} \tag{33}$$

Due to $\mathscr{L}(\kappa+1) > 0$, it can be easily derived that

$$\mathbb{S}_\kappa < \sum_{t=0}^{\kappa} \mathbb{E}\{e^T(t)e(t) - \eta^2 W_F^T(t)W_F(t) + \Delta\mathscr{L}(t)\}. \tag{34}$$

Based on (22), (23), and (31), we obtain

$$\begin{aligned}&\mathbb{E}\{e^T(t)e(t) - \eta^2 W_F^T(t)W_F(t) + \Delta\mathscr{L}(t)\}\\ &= \begin{bmatrix}\hat{E}_F(t)\\ W_F(t)\end{bmatrix}^T \underbrace{\begin{bmatrix}\mathscr{U}_1(t+1) & *\\ \mathscr{U}_2(t+1) & \mathscr{U}_3(t+1)\end{bmatrix}}_{\mathscr{U}(t+1)} \begin{bmatrix}\hat{E}_F(t)\\ W_F(t)\end{bmatrix}\end{aligned} \tag{35}$$

where

$$\begin{aligned}\mathscr{U}_1(t+1) &= \widetilde{M}^T(t+1)\Xi_{h_{t+1}}\widetilde{M}(t+1) - \Xi_{h_t} + \Omega^T(t)\Omega(t)\\ &\quad + \bar{\alpha}^2\widetilde{N}^T(t+1)\Xi_{h_{t+1}}\widetilde{N}(t+1)\\ \mathscr{U}_2(t+1) &= \widetilde{G}^T(t+1)\Xi_{h_{t+1}}\widetilde{M}(t+1)\\ &\quad + \bar{\alpha}^2\widetilde{H}^T(t+1)\Xi_{h_{t+1}}\widetilde{N}(t+1)\\ \mathscr{U}_3(t+1) &= \widetilde{G}^T(t+1)\Xi_{h_{t+1}}\widetilde{G}(t+1) - \eta^2 I\\ &\quad + \bar{\alpha}^2\widetilde{H}^T(t+1)\Xi_{h_{t+1}}\widetilde{H}(t+1).\end{aligned}$$

According to (34) and (35), $\mathscr{U}(t+1) < 0$ will lead to $\mathbb{S}_\kappa < 0$. Using the Schur's complement lemma, $\mathscr{U}(t+1) < 0$ implies that

$$\begin{bmatrix}-\Xi_{h_t} & * & * & * & *\\ 0 & -\eta^2 I & * & * & *\\ \widehat{M}(t+1) & \widehat{G}(t+1) & -\Xi_{h_{t+1}} & * & *\\ \widehat{N}(t+1) & \widehat{H}(t+1) & 0 & -\Xi_{h_{t+1}} & *\\ \Omega(t) & 0 & 0 & 0 & -I\end{bmatrix} < 0 \tag{36}$$

where

$$\begin{aligned}\widehat{M}(t) &= \Xi_{h_t}\widetilde{M}(t), \quad \widehat{G}(t) = \Xi_{h_t}\widetilde{G}(t)\\ \widehat{N}(t) &= \bar{\alpha}\Xi_{h_t}\widetilde{N}(t), \quad \widehat{H}(t) = \bar{\alpha}\Xi_{h_t}\widetilde{H}(t).\end{aligned}$$

Let

$$\begin{aligned}O_i(t) &= \mathrm{diag}\left\{\frac{\Gamma_{i1}(t)}{\xi_{i1}}, \ldots, \frac{\Gamma_{im}(t)}{\xi_{im}}\right\}\\ O(t) &= \mathrm{diag}\{O_1(t), \ldots, O_L(t)\}\\ \widetilde{O}(t) &= \mathrm{diag}\{O(t), O(t)\}.\end{aligned}$$

Then, combining the definitions in Theorem 1, (36) can be converted to

$$\begin{aligned}&\begin{bmatrix}-\Xi_{h_t} & * & * & * & *\\ 0 & -\eta^2 I & * & * & *\\ D(t+1) & P(t+1) & -\Xi_{h_{t+1}} & * & *\\ \bar{\alpha}R(t+1) & \bar{\alpha}S(t+1) & 0 & -\Xi_{h_{t+1}} & *\\ \Omega(t) & 0 & 0 & 0 & -I\end{bmatrix}\\ &+ \begin{bmatrix}0 & 0\\ 0 & 0\\ -\Phi(t+1) & 0\\ 0 & -\bar{\alpha}\Phi(t+1)\\ 0 & 0\end{bmatrix}\widetilde{O}(t+1)\\ &\times \begin{bmatrix}Z\vec{\mathscr{E}}(t) & \alpha Z\Psi_{h_t} & 0 & 0 & 0\\ Z\vec{\mathscr{F}}(t) & Z\Psi_{h_t} & 0 & 0 & 0\end{bmatrix}\\ &+ \begin{bmatrix}Z\vec{\mathscr{E}}(t) & \alpha Z\Psi_{h_t} & 0 & 0 & 0\\ Z\vec{\mathscr{F}}(t) & Z\Psi_{h_t} & 0 & 0 & 0\end{bmatrix}^T \widetilde{O}^T(t+1)\\ &\times \begin{bmatrix}0 & 0\\ 0 & 0\\ -\Phi(t+1) & 0\\ 0 & -\bar{\alpha}\Phi(t+1)\\ 0 & 0\end{bmatrix}^T < 0.\end{aligned} \tag{37}$$

There is no doubt that $\Gamma_i(t) = O_i(t)Z_i$. Because of $|\Gamma_{ij}| \leq \xi_{ij}$, it can be inferred that $\widetilde{O}^T(t)\widetilde{O}(t) \leq I$. In terms of Lemma 1, there is a positive scalar τ such that (37) is equivalent to the following equation:

$$\begin{bmatrix} -\Xi_{h_t} & * & * & * & * \\ 0 & -\eta^2 I & * & * & * \\ D(t+1) & P(t+1) & -\Xi_{h_{t+1}} & * & * \\ \bar{\alpha}R(t+1) & \bar{\alpha}S(t+1) & 0 & -\Xi_{h_{t+1}} & * \\ \Omega(t) & 0 & 0 & 0 & -I \end{bmatrix} + \tau^{-1} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\Phi(t+1) & 0 \\ 0 & -\bar{\alpha}\Phi(t+1) \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\Phi(t+1) & 0 \\ 0 & -\bar{\alpha}\Phi(t+1) \\ 0 & 0 \end{bmatrix}^T + \tau \begin{bmatrix} Z\vec{\mathscr{E}}(t) & \alpha Z\Psi_{h_t} & 0 & 0 & 0 \\ Z\vec{\mathscr{F}}(t) & Z\Psi_{h_t} & 0 & 0 & 0 \end{bmatrix}^T \times \begin{bmatrix} Z\vec{\mathscr{E}}(t) & \alpha Z\Psi_{h_t} & 0 & 0 & 0 \\ Z\vec{\mathscr{F}}(t) & Z\Psi_{h_t} & 0 & 0 & 0 \end{bmatrix} < 0. \tag{38}$$

Applying the Schur's complement lemma to (38), the inequality (25) in Theorem 1 can be obtained. That is the end of the proof. ■

Appendix B
Proof of Theorem 2

Define $\Pi_i(t) = \Xi_{h_t}^{i1} K_i(t)$, and (26) in Theorem 2 can be obtained from (25) in Theorem 1. In addition, according to $\Pi_i(t) = \Xi_{h_t}^{i1} K_i(t)$, it can be easily derived that $K_i(t) = (\Xi_{h_t}^{i1})^{-1}\Pi_i(t)$, which is (30). That is the end of the proof. ■

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