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Event-triggered decentralized security control for large-scale semi-Markovian switching systems with actuator saturation and stochastic network attacks

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ABSTRACT

This work studies the observer-based decentralized event-triggered control for a class of large-scale semi-Markovian switching systems with actuator saturation and network attacks. Taking into account the completely open transmission network environment, Bernoulli random variables are used to model hybrid stochastic time-varying network attacks. Meanwhile, an adaptive event-triggered communication mechanism is investigated to make reasonable use of the limited channel resources. Considering the effects of actuator saturation and network attacks, the observer-based event-triggered control system under hybrid network attacks is established. Furthermore, Lyapunov stability theory is utilized to ensure the stochastic stability of the closed-loop error system with H_∞ performance under hybrid network attack, and the corresponding estimation and control design algorithms have also been derived by solving matrix inequalities. Finally, the experimental simulation results of a real system indicate that the decentralized control method can availably realize the stability of the large-scale system under network attacks.

1. Introduction

Large-scale system is a typical of dynamic systems, which has several subsystems with multiple input-output variables and exhibits certain coupled characteristics. This type of system is widely adopted in engineering domains such as power generation, digital communications, and chemical processing. In recent years, decentralized control strategies have become an attractive control method for dealing with large-scale complex systems [1–4]. The greater flexibility, scalability, and dependability of decentralized control over centralized control are some of its special advantages [5]. However, designing a reliable controller based on a decentralized approach will be a challenge because of the dual complexity of the external network environment and the interconnections among internal subsystems.

On the other hand, due to the randomly occurring environmental changes such as component failures and repairs of components, many dynamic systems experience random variations in practical applications. Markovian jump systems(MJSs) can accurately describe the above behavior of the system, which have been studied extensively over the past few decades [6–9]. Large-scale systems with Markovian jump parameters have also received increasing attention. For example, the authors in [10,11] presented the

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decentralized stabilization results through neighboring mode correlation control methods. The authors in [12] developed decentralized control of power systems based on interconnected Markovian jump models. Compared to MJSs, the sojourn time mechanism of the semi-Markovian jump systems(SMJSs) is more intricate, as it does not simply follow an exponential distribution, but encompasses a wider range of distributions, including the Gaussian and Weibull distributions. However, many conclusions of SMJSs are almost based on the assumption that the transition rates of Markovian chains are perfectly known. In practice, the acquisition of all elements of the transition rate matrix is a challenging task in practical system models, underscoring the importance of investigating different types of transition rates. Recently, there has also been many stability analysis and synthesis methods for SMJSs with generally uncertain transition rates [13–16]. In [17], the authors addressed the problem of reachable set synthesis for a class of nonlinear delayed hidden SMJSs using a security asynchronous controller grounded in the hidden Markov model. In [18], the authors investigated mean-square synchronization of discrete-time dynamic networks with stochastic switching topologies through a semi-Markov kernel framework.

Unfortunately, such approaches typically rely on the structure or boundaries of uncertainty, which is to some extent existentially conservative. Thus, it is more significant to analyse and study on the control problems for more general SMJSs with partially unknown transition rates. The techniques typically employed in MJSs to deal with uncertain and unknown transition rates cannot be directly generalized to SMJSs, which makes it very attractive and challenging to study these problems in large-scale systems with semi-Markovian switching parameters.

The problem of actuator saturation appears in many practical control systems, thus saturation nonlinearity is unavoidable in most actuators. Once an actuator reaches its input limit, it becomes saturated, and any further increase in the input fails to produce a corresponding change in output. The presence of saturation can degrade performance and even lead to closed system instability. Recently, there has been heightened interest in the robust control of nonlinear systems affected by actuator saturation [19–25]. Despite notable advances in this area and encouraging results achieved through control strategies based on semi-Markovian jump models, the control for large-scale SMJSs subject to actuator saturation still remains a challenging problem, which the first motivation for this paper.

It is commonly known that frequent subsystem interactions can squander energy resources and generate network congestion. In order to prevent energy waste, event-triggered sampling was introduced in [26–33] in light of the limited resources. In contrast to the aforementioned static event-triggering schemes, the authors in [31] offered the dynamic ETM with adaptive and variable thresholds for the a class of large-scale systems. The authors in [28] discussed the ETM with continuous triggering threshold. The authors in [32,33] examined distributed event-based security control with partial unknown transfer probabilities and proposed a novel ETM with semi-Markovian switching topology for multi-agent systems. To further reduce computational and transmission resources, an event-triggered scheme with an adaptive law is developed in the current study.

It is notice that the susceptibility of transmittal control signals to malicioius attacks has inspired scholars' interest and prompted a lot of research in recent decade [34–40]. Network attacks, as explained in [34], can be classified as sequential topology attacks, denial-of-service (DoS) attacks and false-data injection (FDI) attacks and so on. Facing with various network attacks, researchers are more concerned about how to design defensive controllers and guarantee the control performance for the real networked industrial system under network attacks [35]. In [36], the impulsive control strategy in multi-agent systems subjected to deception attacks was examined. In [37], the issue of load frequency control under bandwidth constraints and deception attacks has been investigated. In [38], the development of security filters for uncertain stochastic nonlinear systems vulnerable to various network attacks were described. In light of the aforementioned literature, our objective is to create an event-triggering security control strategy for large-scale SMJSs that are vulnerable to network attacks, which is the second motivation for this paper.

In fact, system states are usually not measurable or even unavailable in real systems due to limitations in cost and technology. The observers design becomes an important issue in control systems, since observers can efficiently estimate unavailable states [41]. Considering that many systems actually involve completely or partially unmeasurable state variables, observer-based nonparallel distributed compensation control strategies have been widely used in many practical applications [42]. In addition, observer-based safety control for nonlinear SMJSs has become increasingly prevalent in the past decades and has attracted continuous attention [43–45]. For large-scale SMJSs, the principal challenges in observer-based controller design arise from the necessity to decouple the interconnection terms among subsystems and to manage partially unknown transition probabilities. These challenges often limit the effectiveness of existing approaches in practice. Accordingly, the observer-based control design and performance analysis for large-scale SMJSs in the network environment is still an open issue, which is worth further exploration as the third motivation.

Inspired by the mentioned ideas, we intend developing an observer-based decentralized security control approach for large-scale SMJSs with actuator saturation and network attacks. The three main contributions of this paper are summarized as follows: (i) In comparison to the recent work [32] on decentralized control, both actuator failure and actuator saturation are considered simultaneously in the control input, which is closer to the real system. And a novel observer-based decentralized control model is first proposed for large-scale SMJSs to encounter actuator saturation and the stochastic network attacks imposed on the communication network. (ii) Different from the existing event-triggered methods [2,35,46], an adaptive event-triggering scheme is adopted for each subsystems to alleviate the transmission burden of the large-scale systems. Specifically, a dynamic triggering threshold function is given to adjust automatically according to the changes of the triggering errors. (iii) Fully considering several scenarios of partially unknown transition probabilities, we establish novel mode-dependent stability criteria for the considered system using Lyapunov functional methods and stochastic analysis techniques. Based on these results, both controller and observer gains ensuring the desired H_∞ performance are derived simultaneously.

2. Problem formulation

Consider the following large-scale SMJSs with actuator saturation, which are composed of N coupled subsystems.

$$\begin{cases} \dot{q}_i(t) = \mathcal{A}_i(r_t)q_i(t) + \sum_{j=1,j\neq i}^{N} \mathcal{F}_{ij}(r_t)q_j(t) + \mathcal{B}_i(r_t)\sigma(u_i(t)) + \mathcal{G}_i(r_t)\omega_i(t), \\ y_i(t) = \mathcal{C}_i(r_t)q_i(t), \\ z_i(t) = \mathcal{E}_i(r_t)q_i(t) + \mathcal{D}_i(r_t)\omega_i(t), \end{cases} \tag{1}$$

where $q_i(t)$, $y_i(t)$, $z_i(t)$, $u_i(t)$ present the system state, measured output, the controlled output and controlled input, respectively; $\omega_i(t)$ denotes the disturbance input; $\mathcal{F}_{ij}(r_t)$ is the interconnection matrix between the ith and jth subsystem. $r(t)$ is driven by a continuous-time semi-Markov process over a finite set $S = \{1, 2, \dots, \varrho\}$. The communication probabilities of semi-Markovian process are defined as below:

$$\mathbf{Pr}\{r_{t+\Delta} = k | r_t = m\} = \begin{cases} \pi^i_{mk}(h)\Delta + o(\Delta), m \neq k, \\ 1 + \pi^i_{mm}(h)\Delta + o(\Delta), m = k, \end{cases} \tag{2}$$

where $\Delta > 0, \lim_{\Delta\to 0} o(\Delta)/\Delta = 0$ and $\pi^i_{mk}(h)$ is the transition rate from mth mode at time t to kth mode at time $t+\Delta$, and satisfies $\pi^i_{mm}(h) = -\Sigma^{\varrho}_{k=1,k\neq m}\pi^i_{mk}(h)$. For each mode $r_t = m \in S$, the parameter matrix of the system can be shortened accordingly as $A_{im}, F_{ijm}, B_{im}, G_{im}, C_{im}, E_{im}, D_{im}$.

Consider the case where part of the transition probability matrix is unknown, which indicates the transition probabilities of semi-Markovian process are suppose to be completely unobservable or imprecisely known constrained within defined lower and upper bounds. For example, the mode transition probability matrix can be read as:

$$\begin{bmatrix} \pi^i_{11}(h) & ? & ? & \cdots & \pi^i_{1\varrho}(h) \\ ? & ? & \pi^i_{23}(h) & \cdots & ? \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ ? & \pi^i_{\varrho 2}(h) & \pi^i_{\varrho 3}(h) & \cdots & \pi^i_{\varrho\varrho}(h) \end{bmatrix}, \tag{3}$$

where ? represents a completely unknown element, $\pi^i_{mk}(h) \in [\underline{\pi}^i_{mk}, \overline{\pi}^i_{mk}]$. $\underline{\pi}^i_{mk}$ and $\overline{\pi}^i_{mk}$ are the upper bound and lower bound of the uncertain transition probability respectively. The set Λ is abbreviated as $\Lambda = \Lambda^m_k \cup \Lambda^m_{uk}$, and $\Lambda^m_k = \{k \in S : \pi^i_{mk}(h)$ has the known upper bound and lower bound$\}$, $\Lambda^m_{uk} = \{k \in S : \pi^i_{mk}(h)$ is totally unknown$\}$.

For the purpose of saving network resources, an adaptive event-triggering mechanism that can dynamically adjust the trigger threshold is introduced between the sensor and the observer of the i subsystem. $\hbar$ represents the sampling period, and the release time of the current sampled data, denoted by $t^i_{k+1}\hbar$,is determined by the event trigger condition defined below:

$$t^i_{k+1}\hbar = t^i_k\hbar + \min_{l\in N_l}\left\{l\hbar \mid e^{\mathrm{T}}_{ki}(t)J_i e_{ki}(t) > \eta_i(t)y^{\mathrm{T}}_i(t^i_{k+1}\hbar)J_i y_i(t^i_{k+1}\hbar)\right\}, \tag{4}$$

where $J_i > 0$ is the weighting parameter to be designed; $e_{ki}(t) = y_i(t^i_k\hbar) - y_i(t^i_k\hbar + l\hbar)$ represents the error of the current sampling signal and the latest released signal, $l \in N_l = \{1, 2, \dots, l_M\}$ and $l_M = t^i_{k+1}\hbar - t^i_k\hbar - 1, \eta_i(t)$ is the adaptive event-triggering threshold that satisfies

$$\dot{\eta}_i(t) = \frac{1}{\eta_i(t)}\left(\frac{1}{\eta_i(t)} - \delta_i\right)e^{\mathrm{T}}_{ki}(t)J_i e_{ki}(t), \tag{5}$$

where $\eta_i(0) \in (0, 1]$, and $\delta_i > 0$ adjusts the release frequency of the sampling signal. (Fig. 1)

We divide the interval $[t^i_k\hbar + \tau^i_k, t^i_{k+1}\hbar + \tau^i_{k+1})$ as $l_M + 1$ parts, denoted as $[t^i_k\hbar + \tau^i_k, t^i_{k+1}\hbar + \tau^i_{k+1}) = \bigcup_{l=0}^{l_M} Y_l$, where τ^i_k is the delay of the trigger signal during transmission. Each segment is expressed as $Y_l = [t^i_k\hbar + l\hbar + \tau^i_k, t^i_{k+1}\hbar + l\hbar + \hbar + \tau^i_{k+1})$. For $t \in \Omega_l$, we define $\tau_i(t) = t - t^i_k\hbar + l\hbar$, satisfying the following condition:

$$0 \le \tau^i_k \le \tau_i(t) \le \tau^i_k\hbar + l\hbar \le \tau^i_M, t \in Y_l. \tag{6}$$

Furthermore, $y_i(t^i_k\hbar)$ can be equivalent to

$$y_i(t^i_k\hbar) = e_{ki}(t) + y_i(t - \tau_i(t)), t \in Y_l. \tag{7}$$

Remark 1. In conventional time-triggered strategies, signals are dispatched at uniform intervals irrespective of the system's actual dynamics. Although this ensures the regularity of sampling, it frequently results in superfluous transmissions, thereby wasting bandwidth and energy. To address this inefficiency, event-triggered approaches have been developed, transmitting data only when predefined conditions are satisfied, thus markedly alleviating communication burden. In addition, based on the sampled-data AETM (4), the minimum trigger interval is $\bar{h}$, thereby Zeno behavior cannot occur. Relative to the constant-threshold event-triggered scheme in [26–28], the proposed AETM achieves superior efficiency by employing a state-dependent, time-varying threshold $\eta_i(t)$. Importantly, when $\eta_i(t) = 0$, the AETM naturally degenerates into the conventional time-triggered mechanism. In this work, the AETM (4) is incorporated to substantially reduce communication overhead while simultaneously improving the efficiency of network resource utilization.

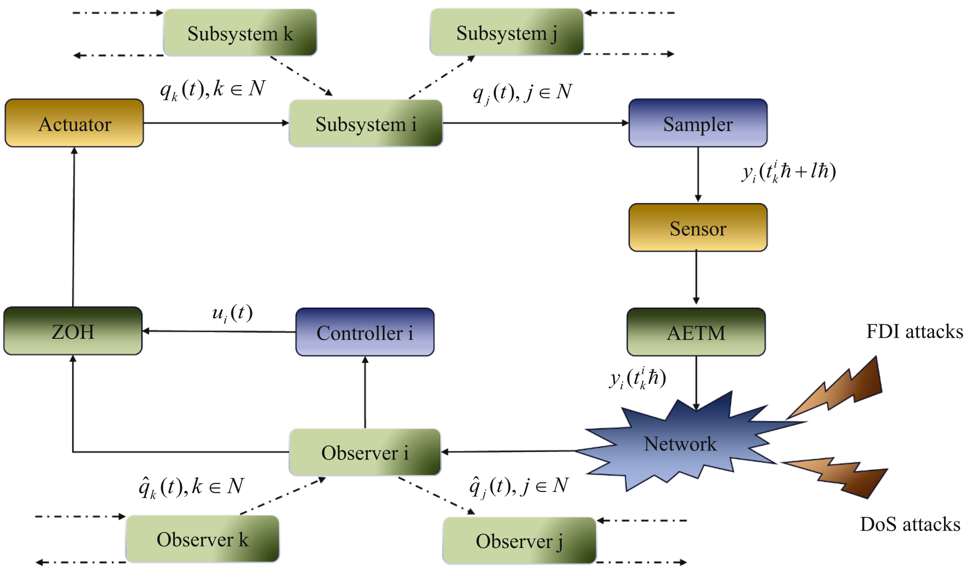

Fig. 1. The structure of large-scale SMJSs.

$\sigma(\cdot)$ in system (1) is the standard saturation function, which is defined as below

$$\sigma(u_i(t)) = [sat(u_{i1}(t)), sat(u_{i2}(t)), \ldots, sat(u_{ip_i}(t))]^{\mathrm{T}}, \tag{8}$$

$$sat(u_i(t)) = \begin{cases} sign(u_i(t))\bar{u}_i, & |\, u_{iv_i}(t) \,| \geqslant \bar{u}_{iv_i}, \\ u_i(t), & |\, u_{iv_i}(t) \,| < \bar{u}_{iv_i}, \end{cases} \tag{9}$$

where $v_i \in \mathcal{I}[1, p_i], u_i(t) = [u_{i1}(t), u_{i2}(t), \ldots, u_{ip_i}(t)]^{\mathrm{T}}$ is the control input for the ith subsystem, and $\bar{u}_i$ indicates saturation level of the $sat(u_{iv_i}(t))$.

Remark 2. The saturation function $\sigma(\cdot)$ defined in (8) and (9) adopts the standard component-wise form, in which each input signal $u_{iv_i}(t)$ is restricted to the admissible interval $[-\bar{u}_{iv_i}, \bar{u}_{iv_i}]$. This choice is motivated by two considerations. First, it accurately characterizes the inherent physical constraints of actuators and thus avoids unachievable control commands. Second, the bounded and continuous nature of the mapping facilitates Lyapunov-based analysis and ensures that the derived stability conditions remain tractable.

Assume that the unexpected actuator failure is independent and random, the random actuator failure model is constructed as follows:

$$u_i^F(t) = \Xi_i \sigma(u_i(t)). \tag{10}$$

In order to facilitate, we define $T_{v_i} = diag\{\underbrace{0, \ldots, 0}_{v_i}, 1, \underbrace{0, \ldots, 0}_{p_i - v_i}\}$, then the control input affected by actuator faults can be reformulated as:

$$u_i^F(t) = \sum_{v_1=1}^{p_i} \chi_{iv_i} T_{v_i} \sigma(u_i(t)), \tag{11}$$

where $\Xi_i = diag\{\chi_{i1}, \chi_{i2}, \ldots, \chi_{ip_i}\}, \chi_{iv_i}$ represents the failure scale factor of each channel. The mathematical expectation of χ_{iv_i} is $\overline{\chi}_{iv_i}$. For convenience, we denote $\overline{\Xi}_i = diag\{\overline{\chi}_{i1}, \overline{\chi}_{i2}, \ldots, \overline{\chi}_{ip_i}\}$.

Because of the openness of communication channels, malicious network attacks and interference often occur randomly during data transmission. In this paper, the effects of various network attacks are considered. Spoofing signals $f_i(y_i(t))$ can replace normal data when the network suffers from deception attacks. Meanwhile, we account for the impact of DoS attacks, thereby illustrating the actual output signals received by the observer through the network as follows:

$$\tilde{y}_i(t) = \alpha_i(t)[\beta_i(t) y_i(t_k^i \hbar) + (1 - \beta_i(t)) f_i(y_i(t))]. \tag{12}$$

Bernoulli variables $\alpha_i(t), \beta_i(t) \in \{0, 1\}$ describe whether DoS attacks or deception attacks occur and the following conditions are met: $\mathbb{E}\{\alpha_i(t)\} = \overline{\alpha}_i$, $\mathbb{E}\{\beta_i(t)\} = \overline{\beta}_i$, $\mathbb{E}\{(\alpha_i(t) - \overline{\alpha}_i)^2\} = s_i^2$, $\mathbb{E}\{(\beta_i(t) - \overline{\beta}_i)^2\} = \iota_i^2$, where $s_i, \iota_i \in [0, 1]$ denote two probability constants.

Remark 3. The measurement output expressed in Eq. (12) includes the following three cases. If $\alpha_i(t) = \beta_i(t) = 1$, then the observer receives $y_i(t_{k+1}^i \hbar)$ at the latest data-discharging instant, which indicates that there are no attacks. $\alpha_i(t) = 0$ implies DoS attacks occurred. The subsystems is suffering from deception attacks $\beta_i(t) = 0$ and $\alpha_i(t) = 1$, then the transmitting data are $\tilde{y}_i(t) = f_i(y_i(t))$.

Consider the influence of random network attack, actuator failure and adaptive trigger constraint on system output, and the following form of state estimation model is established for the ith sub-system of interconnection SMJSs.

$$\begin{cases} \dot{\hat{q}}_i(t) = \mathcal{A}_{im}\hat{q}_i(t) + \sum_{j=1, j\neq i}^{N} \mathcal{F}_{ijm}\hat{q}_j(t) + \mathcal{B}_{im}\Xi_{im}\sigma(u_i^F(t)) - L_{im}(\hat{y}_i(t) - \tilde{y}_i(t)), \\ \hat{y}_i(t) = \mathcal{C}_{im}\hat{q}_i(t), \\ u_i(t) = K_{im}\hat{q}_i(t), \end{cases} \tag{13}$$

where $\hat{q}_i(t)$ is the estimated state, and $\hat{y}_i(t)$ presents the output of i subsystem. L_{im} represents the designed observer gain matrices and K_{im} indicates the controller gain matrices to be designed.

Define $e_i(t) = q_i(t) - \hat{q}_i(t)$, $\zeta_i(t) = \begin{bmatrix} q_i^{\mathrm{T}}(t) & e_i^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$, $\tilde{\omega}_i(t) = \begin{bmatrix} \omega_i^{\mathrm{T}}(t) & f_i^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$, and an augmented system of ith subsystem can be written as

$$\begin{cases} \dot{\zeta}_i(t) = \tilde{\mathcal{A}}_i\zeta_i(t) + \sum_{j=1, j\neq i}^{N} \tilde{\mathcal{F}}_{ij}\zeta_j(t) + \tilde{\mathcal{B}}_i\sigma(\tilde{K}_i\zeta_i(t)) + (\tilde{\mathcal{Z}}_{1i} + \overline{\mathcal{Z}}_{1i})G_i\zeta_i(t - \tau_i(t)) \\ \qquad + (\tilde{\mathcal{Z}}_{2i} + \overline{\mathcal{Z}}_{2i})\tilde{\omega}_i(t) + (\tilde{\mathcal{Z}}_{3i} + \overline{\mathcal{Z}}_{3i})e_{ki}(t), \\ \tilde{z}_i(t) = \tilde{\mathcal{E}}_i\zeta_i(t) + \tilde{\mathcal{D}}_i\tilde{\omega}_i(t), \end{cases} \tag{14}$$

where

$$\tilde{\mathcal{A}}_i = \begin{bmatrix} \mathcal{A}_{im} + \mathcal{B}_{im}K_{im} & -\mathcal{B}_{im}K_{im}, \\ L_{im}\mathcal{C}_{im} & \mathcal{A}_i - L_{im}\mathcal{C}_{im} \end{bmatrix}, \quad \tilde{\mathcal{F}}_{ij} = \begin{bmatrix} \mathcal{F}_{ijm} & 0 \\ 0 & \mathcal{F}_{ijm} \end{bmatrix}, \quad \tilde{\mathcal{B}}_{\omega i} = \begin{bmatrix} \mathcal{B}_{\omega im} \\ 0 \end{bmatrix}, \quad \tilde{K}_{im} = \begin{bmatrix} 0 \\ \mathcal{B}_{im}\Xi_{im} \end{bmatrix},$$

$$\tilde{\mathcal{Z}}_{1i} + \overline{\mathcal{Z}}_{1i} = \begin{bmatrix} 0 \\ -\bar{\alpha}_i\bar{\beta}_i L_{im}\mathcal{C}_{im} \end{bmatrix} + \begin{bmatrix} 0 \\ m_1(t)L_{im}\mathcal{C}_{im} \end{bmatrix}, \quad \tilde{\mathcal{Z}}_{2i} + \overline{\mathcal{Z}}_{2i} = \begin{bmatrix} \mathcal{G}_{im} & 0 \\ \mathcal{G}_{im} & -\bar{\alpha}_i(1-\bar{\beta}_i)L_{im} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & m_2(t)L_{im} \end{bmatrix},$$

$$\tilde{\mathcal{Z}}_{3i} + \overline{\mathcal{Z}}_{3i} = \begin{bmatrix} 0 \\ \bar{\alpha}_i\bar{\beta}_i L_{im}\mathcal{C}_{im} \end{bmatrix} + \begin{bmatrix} 0 \\ m_1(t)L_{im}\mathcal{C}_{im} \end{bmatrix}, \quad G_i = \begin{bmatrix} I & 0 \end{bmatrix}, \quad \tilde{\mathcal{D}}_i = \begin{bmatrix} \mathcal{D}_{im} & 0 \end{bmatrix},$$

$$m_1(t) = -\alpha_i(t)\beta_i(t) + \bar{\alpha}_i\bar{\beta}_i, \quad m_2(t) = -\alpha_i(t)(1-\beta_i(t)) + \bar{\alpha}_i(1-\bar{\beta}_i), \quad \tilde{\mathcal{E}}_i = \begin{bmatrix} \mathcal{E}_{im} & 0 \end{bmatrix}.$$

In order to facilitate the actuator saturation control design, we pull into a linear subset:

$$\Psi(S_{im}) \triangleq \left\{ \zeta_i(t) \in R^{2ni} : \mid S_{im}\zeta_i(t) \mid \leq \bar{u}_i \right\}, \tag{15}$$

where $S_{im} \in R^{p_i\times 2ni}, \bar{u}_i = [\bar{u}_{i1}, \bar{u}_{i2}, \ldots, \bar{u}_{ip_i}]^{\mathrm{T}}, S_{im}^{v_i}$ is the v_i row of the matrix S_{im}, for any positive matrix P_{im}, a shrinkage invariant ellipsoid is defined as:

$$\Omega(P_{im}, 1) \triangleq \left\{ \zeta_i(t) \in R^{2ni} : \mid \zeta_i^{\mathrm{T}}(t)P_{im}\zeta_i(t) \mid \leq 1 \right\}. \tag{16}$$

We definite μ_i as the set of $p_i \times p_i$ diagonal matrices with diagonal entries either of 1 or 0, $\forall \Gamma_{\vartheta_i} \in \mu$, where $\vartheta_i \in \mathcal{I}[1, 2^{p_i}]$, then definite $\Gamma_{\vartheta_i}^- \triangleq I - \Gamma_{\vartheta_i}$, so $\Gamma_{\vartheta_i}^- \in \mu_i$.

Lemma 1 ([47]). *For the given matrix $\tilde{K}_{im} \in R^{p_i\times 2ni}, S_{im} \in R^{p_i\times 2ni}$, if $\zeta_i(t) \in \Psi(S_{im})$,*

$$\sigma(\tilde{K}_{im}\zeta_i(t)) = \sum_{\vartheta=1}^{2^{p_i}} \theta_{\vartheta_i}(\Gamma_{\vartheta_i}\tilde{K}_{im} + \Gamma_{\vartheta_i}^- S_{ik})\zeta_i(t), \tag{17}$$

where $\sum_{\vartheta=1}^{2^{p_i}} = 1, 0 \leqslant \theta_{\vartheta_i} \leqslant 1, i \in \mathbf{I}[1, n], \tilde{K}_{im} = \begin{bmatrix} 0 & K_{im} \end{bmatrix}$.

Therefore, if $\zeta_i(t) \in \Psi(S_{im})$, (14) will be expressed as:

$$\begin{cases} \dot{\zeta}_i(t) = \sum_{\vartheta_i=1}^{2^{p_i}} \theta_{\vartheta_i}[\bar{\mathcal{A}}_i\zeta_i(t) + \sum_{j=1, j\neq i}^{N} \tilde{\mathcal{F}}_{ij}\zeta_j(t) + (\tilde{\mathcal{Z}}_{1i} + \overline{\mathcal{Z}}_{1i})G_i\zeta_i(t - \tau_i(t)) \\ \qquad + (\tilde{\mathcal{Z}}_{2i} + \overline{\mathcal{Z}}_{2i})\tilde{\omega}_i(t) + (\tilde{\mathcal{Z}}_{3i} + \overline{\mathcal{Z}}_{3i})e_{ki}(t)], \\ \tilde{z}_i(t) = \tilde{\mathcal{E}}_i\zeta_i(t) + \tilde{\mathcal{D}}_i\tilde{\omega}_i(t), \end{cases} \tag{18}$$

where

$$\bar{\mathcal{A}}_i = \begin{bmatrix} \mathcal{A}_{im} + \mathcal{B}_{im}K_{im} & -\mathcal{B}_{im}K_{im} \\ L_{im}\mathcal{C}_{im} + \mathcal{B}_{im}\chi_{im}\Gamma_{\vartheta_i}^- S_{im}^a & \mathcal{A}_{im} - L_{im}\mathcal{C}_{im} + \mathcal{B}_{im}\chi_{im}\Gamma_{\vartheta_i}K_{im} + \mathcal{B}_{im}\Xi_{im}\Gamma_{\vartheta_i}^- S_{im}^b \end{bmatrix}, \begin{bmatrix} S_{im}^a & S_{im}^b \end{bmatrix} = S_{im}.$$

Combining (11), $\bar{\mathcal{A}}_i$ can be rewritted as

$$\bar{\mathcal{A}}_i = \begin{bmatrix} \mathcal{A}_{im} + \mathcal{B}_{im}K_{im} & -\mathcal{B}_{im}K_{im} \\ L_{im}\mathcal{C}_{im} + \sum_{v_i=1}^{p_i} \mathcal{B}_{im}\chi_{iv_im}T_{v_i}\Gamma_{\vartheta_i}^- S_{im}^a & \mathcal{A}_{im} - L_{im}\mathcal{C}_{im} + \sum_{v_i=1}^{p_i} \mathcal{B}_{im}\chi_{iv_im}T_{v_i}(\Gamma_{\vartheta_i}K_{im} + \mathcal{B}_{im}\Xi_{im}\Gamma_{\vartheta_i}^- S_{im}^b) \end{bmatrix}.$$

For convenience, we definite the mathematical expectation of $\bar{\mathcal{A}}_i$ as $\bar{\bar{\mathcal{A}}}_i$:

$$\bar{\bar{\mathcal{A}}}_i = \begin{bmatrix} \mathcal{A}_{im} + \mathcal{B}_{im}K_{im} & -\mathcal{B}_{im}K_{im} \\ L_{im}\mathcal{C}_{im} + \sum_{v_i=1}^{p_i} \mathcal{B}_{im}\bar{\chi}_{iv_im}T_{v_i}\Gamma_{\vartheta_i}^- S_{im}^a & \mathcal{A}_{im} - L_{im}\mathcal{C}_{im} + \sum_{v_i=1}^{p_i} \mathcal{B}_{im}\chi_{iv_im}T_{v_i}(\Gamma_{\vartheta_i}K_{im} + \mathcal{B}_{im}\Xi_{im}\Gamma_{\vartheta_i}^- S_{im}^b) \end{bmatrix}.$$

Definition 1 ([48]). For the given initial condition $(\zeta_i(0), r_0)$, if there exists positive definite scalar $\mathcal{F}(\zeta_i(0), r_0)$ satisfing

$$\mathbb{E}\left\{ \int_0^\infty \zeta_i^{\mathrm{T}}(t)\zeta_i(t)dt|(\zeta_i(0), r_0) \right\} \leqslant \mathcal{F}(\zeta_i(0), r_0), \tag{19}$$

the closed loop system (14) is stochastically stable.

Definition 2 ([48]). For a positive scalar $\gamma > 0$, if the condition(19) is satisfied when $\tilde{\omega}_i(t) \equiv 0$, the SMJSs (14) exhibits stochastic stability with H_∞ performance when the subsequent inequality holds with any nonzero $\tilde{\omega}(t)$:

$$\mathbb{E}\left\{ \int_0^\infty \tilde{z}^{\mathrm{T}}(t)\tilde{z}(t)dt \right\} \leq \gamma^2 \mathbb{E}\left\{ \int_0^\infty \tilde{\omega}(t)^{\mathrm{T}}(t)\tilde{\omega}(t)dt \right\}, \tag{20}$$

where $\tilde{\omega}(t) = [\tilde{\omega}_1^{\mathrm{T}}(t), \tilde{\omega}_2^{\mathrm{T}}(t), \dots, \tilde{\omega}_N^{\mathrm{T}}(t)]^{\mathrm{T}} \neq 0$ and $\tilde{z}(t) = [\tilde{z}_1^{\mathrm{T}}(t), \tilde{z}_2^{\mathrm{T}}(t), \dots, \tilde{z}_N^{\mathrm{T}}(t)]^{\mathrm{T}}$.

Lemma 2. *[[49]] Given scalar $\kappa_i \in (0,1)$, the continuous functions $\tau_i(t) \in [0, \tau_M^i]$ and the vector $\dot{\zeta}(t) : [-\tau_i(t), 0] \longrightarrow \mathbb{R}_i^n$, there exist the positive definite matrices $R_i > 0$ and the matrices U_i with appropriate dimensions, the following inequalities satisfy*

$$\tau_M^i \left\{ \int_{t-\tau_M^i}^t \dot{\zeta}_i^{\mathrm{T}}(s) G_i^{\mathrm{T}} R_i G_i \dot{\zeta}_i(s) ds \right\} \geqslant \varpi_1^{\mathrm{T}} \tilde{R}_{1i} \varpi_1 + \varpi_2^{\mathrm{T}} \tilde{R}_{2i} \varpi_2 + 2\varpi_1^{\mathrm{T}} U_i \varpi_2, \tag{21}$$

where

$$\begin{aligned}
\tilde{R}_{1i} &= \tilde{R}_i + (1-\kappa_i)(\tilde{R}_i - S_i \tilde{R}_i^{-1} S_i^{\mathrm{T}}), \quad \tilde{R}_{2i} = \tilde{R}_i + \kappa_i(\tilde{R}_i - S_i \tilde{R}_i^{-1} S_i^{\mathrm{T}}), \quad \tilde{R}_i = diag\{R_i, 3R_i\}, \\
\varpi_1 &= \begin{bmatrix} \zeta_i^{\mathrm{T}}(t-\tau_i(t))G_i - \zeta_i^{\mathrm{T}}(t-\tau_M^i)G_i \\ \zeta_i^{\mathrm{T}}(t-\tau_i(t))G_i + \zeta_i^{\mathrm{T}}(t-\tau_M^i)G_i - 2\phi_{1i}^{\mathrm{T}} G_i \end{bmatrix}, \quad \varpi_2 = \begin{bmatrix} \zeta_i^{\mathrm{T}}(t)G_i - \zeta_i^{\mathrm{T}}(t-\tau_M^i)G_i \\ \zeta_2^{\mathrm{T}}(t)G_i + \zeta_i^{\mathrm{T}}(t-\tau_i(t))G_i - 2\phi_{2i}^{\mathrm{T}} G_i \end{bmatrix}, \\
U_i &= \begin{bmatrix} U_{1i} & U_{2i} \\ U_{3i} & U_{4i} \end{bmatrix}, \quad \phi_{1i} = \frac{1}{\tau_i(t)} \int_{t-\tau_M^i}^t \zeta_i(s)ds, \quad \phi_{2i} = \frac{1}{\tau_M^i - \tau_i(t)} \int_{t-\tau_i(t)}^{t-\tau_M^i} \zeta_i(s)ds.
\end{aligned}$$

Lemma 3. *[[50]] For the given the positive definite matrices $X_{im} > 0$, $\tilde{R}_i > 0$ and scalar $\varepsilon_i > 0$, the following inequalities satisfy*

$$-X_{im}\tilde{R}_i^{-1}X_{im} \leqslant -2\varepsilon_i X_{im} + \varepsilon_i^2 \tilde{R}_i, m \in S. \tag{22}$$

3. Main result

In this section, the decentralized observer-based control problem for large-scale systems will be investigated by using the Lyapunov function analysis methodology. In Theorem 1, a sufficient condition for the stochastic asymptotic stability of the system (14) is given. Then, the observer gains and controller gains are derived in Theorem 2 by solving LMIs.

Theorem 1. *For given $\kappa_i \in (0,1), \tau_M^i > 0, 0 < \bar{\alpha}_i < 1, 0 < \bar{\beta}_i < 1, \varepsilon_0 > 0, \gamma > 0$, the augmented closed loop subsystem i in (14) achieves stochastically stable with the guaranteed H_∞ performance for all $\tilde{\omega}_i$, if there exist $P_{im} > 0, \mathcal{M}_{im} > 0, \mathcal{M}_i > 0, R_i > 0, T_i > 0$ and matrices U_{1i}, U_{2i}, U_{3i} and U_{4i} with appropriate dimensions satisfying the following conditions hold:*

$$\Omega(P_{im}, 1) \subset \Psi(S_{im}), \tag{23}$$

(1) If $\Lambda_k^m \neq \emptyset$, and $\Lambda_{uk}^m \neq \emptyset$, $m \in \Lambda_k^m$, for $\forall l \in \Lambda_{uk}^m$,

$$\begin{bmatrix}
\tilde{\Phi}_{im} & * & * & * & * & * \\
\Upsilon_{21} & \Upsilon_{22} & * & * & * & * \\
\Upsilon_{31} & \Upsilon_{32} & -R_i & * & * & * \\
\Upsilon_{41} & 0 & 0 & -R_i & * & * \\
\Upsilon_{51} & 0 & 0 & 0 & \Upsilon_{55} & * \\
\Upsilon_{61} & * & * & * & * & \Upsilon_{66}
\end{bmatrix} < 0, \tag{24}$$

$$-\mathcal{M}_i + \sum\nolimits_{k\in\Lambda_k^m} \pi_{mk}^i (\mathcal{M}_{ik} - \mathcal{M}_{il}) < 0, \tag{25}$$

where

$$\tilde{\Phi}_{im} = \begin{bmatrix}
\tilde{\Phi}_{im}^{11} & * & * & * & * & * & * \\
\Phi_{im}^{21} & \Phi_{im}^{22} & * & * & * & * & * \\
\Phi_{im}^{31} & \Phi_{im}^{32} & \Phi_{im}^{33} & * & * & * & * \\
\Phi_{im}^{41} & \Phi_{im}^{42} & \Phi_{im}^{43} & \Phi_{im}^{44} & * & * & * \\
\Phi_{im}^{51} & \Phi_{im}^{52} & -U_{4i} & U_{4i} & \Phi_{im}^{55} & * & * \\
\tilde{\mathcal{Z}}_{2i}^{\mathrm{T}} P_{im} & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\
\tilde{\mathcal{Z}}_{3i}^{\mathrm{T}} P_{im} & 0 & 0 & 0 & 0 & 0 & -\delta_i J_i
\end{bmatrix} < 0,$$

$$\tilde{\Phi}_{im}^{11} = sym\{P_{im}\bar{\bar{\mathcal{A}}}_i\} + G_i^{\mathrm{T}}\mathcal{M}_{im}G_i + \tau_M^i G_i^{\mathrm{T}}\mathcal{M}_i G_i + \varepsilon_0^{-1} P_{im} - 4(1+\kappa_i)G_i^{\mathrm{T}} R_i G_i + \sum\nolimits_{k\in\Lambda_k^m} \pi_{mk}^i(h)(P_{ik} - P_{il}).$$

(2) If $\Lambda_k^m \neq \varnothing$, and $\Lambda_{uk}^m \neq \varnothing$, $m \in \Lambda_{uk}^m$, for $\forall l \in \Lambda_{uk}^m$, $l \neq m$,

$$\begin{bmatrix} \Phi_{im} & * & * & * & * & * \\ \Upsilon_{21} & \Upsilon_{22} & * & * & * & * \\ \Upsilon_{31} & \Upsilon_{32} & -R_i & * & * & * \\ \Upsilon_{41} & 0 & 0 & -R_i & * & * \\ \Upsilon_{51} & 0 & 0 & 0 & \Upsilon_{55} & * \\ \Upsilon_{61} & * & * & * & * & \Upsilon_{66} \end{bmatrix} < 0, \tag{26}$$

$$P_{im} - P_{il} > 0, \quad \mathcal{M}_{im} - \mathcal{M}_{il} > 0, \tag{27}$$

$$-\mathcal{M}_i + \sum\nolimits_{k\in\Lambda_k^m} \pi_{mk}^i(\mathcal{M}_{ik} - \mathcal{M}_{il}) < 0, \tag{28}$$

where

$$\check{\Phi}_{im} = \begin{bmatrix} \check{\Phi}_{im}^{11} & * & * & * & * & * & * \\ \Phi_{im}^{21} & \Phi_{im}^{22} & * & * & * & * & * \\ \Phi_{im}^{31} & \Phi_{im}^{32} & \Phi_{im}^{33} & * & * & * & * \\ \Phi_{im}^{41} & \Phi_{im}^{42} & \Phi_{im}^{43} & \Phi_{im}^{44} & * & * & * \\ \Phi_{im}^{51} & \Phi_{im}^{52} & -U_{4i} & U_{4i} & \Phi_{im}^{51} & * & * \\ \tilde{\mathcal{Z}}_{2i}^{\mathrm{T}} P_{im} & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\ \tilde{\mathcal{Z}}_{3i}^{\mathrm{T}} P_{im} & 0 & 0 & 0 & 0 & 0 & -\delta_i J_i \end{bmatrix} < 0,$$

$$\check{\Phi}_{im}^{11} = sym\{P_{im}\bar{\tilde{\mathcal{A}}}_i\} + G_i^{\mathrm{T}}\mathcal{M}_{im}G_i + \tau_M^i G_i^{\mathrm{T}}\mathcal{M}_i G_i + \varepsilon_0^{-1}P_{im} - 4(1+\kappa_i)G_i^{\mathrm{T}}R_iG_i + \sum\nolimits_{k\in\Lambda_k^m}\pi_{mk}^i(h)(P_{ik} - P_{il}),$$

$$\Phi_{im}^{21} = \tilde{\mathcal{Z}}_{1i}P_{im} - 2(1+\kappa_i)R_iG_i - U_{1i}G_i - U_{2i}G_i - U_{3i}G_i - U_{4i}G_i,$$

$$\Phi_{im}^{22} = C_{im}^{\mathrm{T}}T_iC_{im} - 2(4+\kappa_i)R_i - 2U_{1i} - 2U_{2i} - 2U_{3i} - 2U_{4i},$$

$$\Phi_{im}^{31} = -U_{1i}G_i - U_{2i}G_i + U_{3i}G_i + U_{4i}G_i,$$

$$\Phi_{im}^{32} = -2(2-\kappa_i)R_i + U_{1i} - U_{2i} - U_{3i} + U_{4i},$$

$$\Phi_{im}^{33} = -\mathcal{M}_{im} - 4(2-\kappa_i)R_i,$$

$$\Phi_{im}^{51} = (3+3\kappa_i)R_iG_i,$$

$$\Phi_{im}^{41} = -U_{3i}G_i - U_{4i}G_i,$$

$$\Phi_{im}^{42} = U_{3i} - U_{4i} + 3(2-\kappa_i)R_i,$$

$$\Phi_{im}^{43} = (6-3\kappa_i)R_i,$$

$$\Phi_{im}^{44} = (-6+3\kappa_i)R_i,$$

$$\Phi_{im}^{52} = -U_{2i} + (3+3\kappa_i)R_i,$$

$$\Phi_{im}^{55} = (-3-3\kappa_i)R_i,$$

$$\Upsilon_{im}^{31} = \tau_M^i R_iG_i\begin{bmatrix}\tilde{\mathcal{A}}_i & \tilde{\mathcal{Z}}_{1i} & 0 & 0 & 0 & \tilde{\mathcal{Z}}_{2i} & \tilde{\mathcal{Z}}_{3i}\end{bmatrix},$$

$$\Upsilon_{im}^{41} = \tau_M^i R_iG_i\begin{bmatrix}0 & s_i\iota_i\bar{\mathcal{Z}}_{1i} & 0 & 0 & 0 & s_i(1-\iota_i)\bar{\mathcal{Z}}_{2i} & s_i\iota_i\bar{\mathcal{Z}}_{3i}\end{bmatrix},$$

$$\Upsilon_{im}^{22} = diag\{-I, -I, -(N-1)^{-1}P_{im}\},$$

$$\Upsilon_{im}^{32} = \begin{bmatrix}0 & 0 & s_i\iota_i\bar{\mathcal{Z}}_{3i}\end{bmatrix},$$

$$\Upsilon_{im}^{55} = diag\{-(1-\kappa_i)^{-1}R_i, -3(1-\kappa_i)^{-1}R_i\},$$

$$\Upsilon_{im}^{66} = diag\{-\kappa_i^{-1}R_i, -3\kappa_i^{-1}R_i\},$$

$$\Upsilon_{im}^{21} = \begin{bmatrix}\tilde{\mathcal{E}}_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tilde{\mathcal{D}}_i & 0 \\ \tilde{\mathcal{F}}_{ij}^{\mathrm{T}}P_{im} & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Upsilon_{im}^{51} = \begin{bmatrix}0 & U_{1i}+U_{3i} & -U_{1i}+U_{3i} & -U_{3i} & 0 & 0 & 0 \\ 0 & U_{2i}+U_{4i} & -U_{2i}+U_{4i} & -U_{4i} & 0 & 0 & 0\end{bmatrix},$$

$$\Upsilon_{im}^{61} = \begin{bmatrix}U_{1i}G_i+U_{3i}G_i & -U_{1i}G_i+U_{3i}G_i & 0 & 0 & -U_{3i} & 0 & 0 \\ U_{2i}G_i+U_{4i}G_i & -U_{2i}G_i+U_{4i}G_i & 0 & 0 & -U_{4i} & 0 & 0\end{bmatrix}.$$

Proof. The set $\bigcap_{m=1}^{s}\Omega(P_{im}, 1)$ contains the domain of attraction in the sense of mean square of the closed loop system (19) under the controller. Suppose $q_i(t) \in \bigcap_{m=1}^{s}\Omega(P_{im}, 1)$, then the (23) can be equivalent to $q_i(t) \in \Psi(S_{im})$. By using Lemma 1, $\sigma(\tilde{K}_{im}\zeta_i(t))$ can be denoted with $\sigma(\tilde{K}_{im}\zeta_i(t)) = \Sigma_{\vartheta_i=1}^{2^{p_i}}\theta_{\vartheta_i}(\Gamma_{\vartheta_i}\tilde{K}_{im} + \Gamma_{\vartheta_i}^{-}S_{ik})\zeta_i(t)$. Define the following Lyapunov-Krasovskii functional

$$V(\zeta(t), r_t) = \sum_{i=1}^{N}\sum_{\kappa=1}^{4} V_\kappa(\zeta_i(t), r_t), \tag{29}$$

where

$$
\begin{aligned}
V_1(\zeta_i(t),r_t) &= \zeta_i^{\mathrm{T}}(t)P_i(r_t)\zeta_i(t),\\
V_2(\zeta_i(t),r_t) &= \int_{t-\tau_M^i}^{t}\zeta_i^{\mathrm{T}}(s)G_i^{\mathrm{T}}\mathcal{M}_i(r_t)G_i\zeta_i(s)ds+\int_{-\tau_M^i}^{0}\int_{t+\theta}^{t}\zeta_i^{\mathrm{T}}(s)G_i^{\mathrm{T}}\mathcal{M}_iG_i\zeta_i(s)dsd\theta,\\
V_3(\zeta_i(t),r_t) &= \tau_M^i\int_{-\tau_M^i}^{0}\int_{t+\theta}^{t}\dot{\zeta}_i^{\mathrm{T}}(s)G_i^{\mathrm{T}}R_iG_i\dot{\zeta}_i(\theta)d\theta,\\
V_4(\xi_i(t),r_t) &= \frac{1}{2}\eta_i^2(t).
\end{aligned}
$$

□

Define $\mathcal{L}$ as the weak infinitesimal operator for stochastic processes $\{\xi_i(t),r_t,t\geqq 0\}$, we have

$$
\begin{aligned}
\mathcal{L}V_1(\zeta_i(t),r_t) &= \lim_{\Delta\to 0^+}\frac{1}{\Delta}\Bigg[\sum_{k=1,k\neq m}^{\varrho}\frac{\lambda_{mk}^i\Big(\aleph_m(h+\delta)-\aleph_m(h)\Big)}{1-\aleph_m(h)}\zeta_i^{\mathrm{T}}(t+\delta)P_{ik}\zeta_i(t+\Delta)\\
&\quad+\frac{1-\aleph_m(h+\Delta)}{1-\aleph_m(h)}\zeta_i(t+\Delta)P_{im}\zeta_i(t+\Delta)-\zeta_i^{\mathrm{T}}P_{im}\zeta_i(t)\Bigg]\\
&= \lim_{\Delta\to 0^+}\frac{1}{\Delta}\Bigg[\sum_{k=1,k\neq m}^{\varrho}\frac{\lambda_{mk}^i\Big(\aleph_m(h+\Delta)-\aleph_m(h)\Big)}{1-\aleph_m(h)}\zeta_i^{\mathrm{T}}(t+\Delta)P_{ik}\zeta_i(t+\Delta)\\
&\quad+\frac{1-\aleph_m(h+\Delta)}{1-\aleph_m(h)}\Big(\zeta_i(t+\Delta)-\zeta_i(t)\Big)P_{im}\zeta_i(t+\Delta)\\
&\quad+\frac{1-\aleph_m(h+\Delta)}{1-\aleph_m(h)}\zeta_i(t)P_{im}\Big(\zeta_i(t+\Delta)-\zeta_i(t)\Big)-\frac{\aleph_m(h+\Delta)-\aleph_m(h)}{1-\aleph_m(h)}\zeta_i^{\mathrm{T}}P_{im}\zeta_i(t)\Bigg],
\end{aligned}
\tag{30}
$$

where h represents residence time when the systems jump from the preceding mode to mode m; $\aleph_m(h)$ is time cumulative distribution function of the residence time when systems are in mode m; λ_{mk}^i denotes the probability density from mode m to mode k. By applying the property of the cumulative distribution function, we can get:

$$
\lim_{\Delta\to 0^+}\frac{1-\aleph_m(h+\Delta)}{1-\aleph_m(h)}=1,\quad \lim_{\Delta\to 0^+}\frac{\aleph_m(h+\Delta)-\aleph_m(h)}{1-\aleph_m(h)}=\pi_m^i(h).
\tag{31}
$$

where $\pi_m^i(h)$ denotes the probability density in the mode m, when $m\neq k$, we have $\pi_{mk}^i(h)=\lambda_{mk}^i\pi_m^i(h)$ and $\pi_{mm}^i(h)=-\sum_{k=1,k\neq m}^{\varrho}\pi_{mk}^i(h)$. Suppose that there exit scalar $\varepsilon_{ij}^{-1}\in(0,\varepsilon_0^{-1}]$, then one can get

$$
2\zeta_i^{\mathrm{T}}(t)P_{im}\sum_{j=1,j\neq i}^{N}\tilde{\mathcal{F}}_{ij}(r_t)\zeta_j(t)\leqslant\varepsilon_o^{-1}\zeta_i^{\mathrm{T}}(t)P_{im}\zeta_i(t)+(N-1)\sum_{j=1,j\neq i}^{N}\varepsilon_{ij}\zeta_j^{\mathrm{T}}(t)\tilde{\mathcal{F}}_{ij}^{\mathrm{T}}P_{ij}\tilde{\mathcal{F}}_{ij}\zeta_j(t).
$$

For ε_{ij} is a smaller positive number, thus $\varepsilon_{ij}P_{ij}<P_{im}(j\neq m)$ is relatively easy to implement. We can convert the formula to

$$
2\zeta_i^{\mathrm{T}}(t)P_{im}\sum_{j=1,j\neq i}^{N}\tilde{\mathcal{F}}_{ij}(r_t)\zeta_j(t)\leqslant\varepsilon_o^{-1}\zeta_i^{\mathrm{T}}(t)P_{im}\zeta_i(t)+(N-1)\sum_{j=1,j\neq i}^{N}\zeta_j^{\mathrm{T}}(t)\tilde{\mathcal{F}}_{ij}^{\mathrm{T}}P_{im}\tilde{\mathcal{F}}_{ij}\zeta_j(t).
\tag{32}
$$

Similarly, one can obtain that

$$
\begin{aligned}
\mathcal{L}V_2(\zeta_i(t),r_t) &= \zeta_i^{\mathrm{T}}(t)G_i^{\mathrm{T}}\mathcal{M}_{im}G_i\zeta_i(t)-\zeta_i^{\mathrm{T}}(t-\tau_M^i)G_i^{\mathrm{T}}\mathcal{M}_{im}G_i\zeta_i(t-\tau_M^i)+\tau_M^i\zeta_i^{\mathrm{T}}(t)G_i^{\mathrm{T}}\mathcal{M}_{im}G_i\zeta_i(t)\\
&\quad+\int_{t-\tau_M^i}^{t}\zeta_i^{\mathrm{T}}(t)G_i^{\mathrm{T}}\Big(-\mathcal{M}_i+\sum_{k=1}^{q}\pi_{mk}^i(h)\mathcal{M}_{ik}\Big)G_i\zeta_i(s)ds,\\
\mathcal{L}V_3(\zeta_i(t),r_t) &= (\tau_M^i)^2\dot{\zeta}_i^{\mathrm{T}}(t)G_i^{\mathrm{T}}R_iG_i\dot{\zeta}_i(t)-\tau_M^i\int_{t-\tau_M^i}^{t}\dot{\zeta}_i^{\mathrm{T}}(t)G_i^{\mathrm{T}}R_iG_i\dot{\zeta}_i(s)ds,\\
\mathcal{L}V_4(\zeta_i(t),r_t) &= \eta_i(t)\dot{\eta}_i(t)=\Big(\frac{1}{\eta_i(t)}-\delta_i\Big)e_{ki}(t)^{\mathrm{T}}J_ie_{ki}(t)\\
&\leq\zeta_i^{\mathrm{T}}(t-\tau_i)G_i^{\mathrm{T}}C_{im}^{\mathrm{T}}J_iC_{im}G_i\zeta_i(t-\tau_i)-\delta_ie_{ki}(t)^{\mathrm{T}}J_ie_{ki}(t).
\end{aligned}
\tag{33}
$$

By using Lemma 2, we get

$$
-\tau_M^i\int_{t-\tau_M^i}^{t}\dot{\zeta}_i^{\mathrm{T}}(t)G_i^{\mathrm{T}}R_iG_i\dot{\zeta}_i(s)ds\xi_i(t)^{\mathrm{T}}\hat{\Phi}_{im}\xi_i(t)+(1-\kappa_i)\varpi_1^{\mathrm{T}}U_i\tilde{R}_i^{-1}U_i^{\mathrm{T}}\varpi_1+\kappa_i\varpi_2^{\mathrm{T}}U_i\tilde{R}_i^{-1}U_i^{\mathrm{T}}\varpi_2,
\tag{34}
$$

where

$$\xi_i(t) = \begin{bmatrix}\zeta_i^{\mathrm{T}}(t) & \zeta_i^{\mathrm{T}}(t-\tau_i(t))G_i & \zeta_i^{\mathrm{T}}(t-\tau_M^i)G_i & 2\phi_{1i}^{\mathrm{T}}G_i & 2\phi_{2i}^{\mathrm{T}}G_i\end{bmatrix},$$

$$\hat{\Phi}_{im} = \begin{bmatrix} -4(1+\kappa_i)G_i^{\mathrm{T}}R_iG_i & * & * & * & * \\ \hat{\Phi}_{im}^{21} & \hat{\Phi}_{im}^{22} & * & * & * \\ \Phi_{im}^{31} & \Phi_{im}^{32} & \hat{\Phi}_{im}^{33} & * & * \\ \Phi_{im}^{41} & \Phi_{im}^{42} & \Phi_{im}^{43} & \Phi_{im}^{44} & * \\ \Phi_{im}^{51} & \Phi_{im}^{52} & -U_{4i} & U_{4i} & \Phi_{im}^{55} \end{bmatrix},$$

$$\hat{\Phi}_{im}^{21} = -2(1+\kappa_i)R_iG_i - U_{1i}G_i - U_{2i}G_i - U_{3i}G_i - U_{4i}G_i,$$

$$\hat{\Phi}_{im}^{22} = -2(4+\kappa_i)R_i - 2U_{1i} - 2U_{2i} + 2U_{3i} + 2U_{4i}, \quad \hat{\Phi}_{im}^{33} = -4(2-\kappa_i)R_i.$$

Notice that if $-\mathcal{M}_i + \sum_{k=1}^{\varrho} \pi_{mk}^i(h)\mathcal{M}_{ik} < 0$, then by combing (30)–(34) and using Schur complementary lemma, we can get

$$\mathbb{E}\left\{\mathcal{L}V(\zeta_i(t),r_t) + \tilde{z}_i^{\mathrm{T}}(t)\tilde{z}_i(t) - \gamma^2\tilde{\omega}_i^{\mathrm{T}}(t)\tilde{\omega}_i(t)\right\} \leqslant \mathbb{E}\left\{\sum_{i=1}^{N} \Psi_i^{\mathrm{T}}(t)\Upsilon_{im}\Psi_i(t)\right\},$$

with

$$\Psi_i^{\mathrm{T}}(t) = \begin{bmatrix}\xi_i^{\mathrm{T}}(t) & \tilde{\omega}_i^{\mathrm{T}}(t) & e_{k_i}^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}},$$

$$\Upsilon_{im} = \begin{bmatrix} \Phi_{im} & * & * & * & * & * \\ \Upsilon_{21} & \Upsilon_{22} & * & * & * & * \\ \Upsilon_{31} & \Upsilon_{32} & -R_i & * & * & * \\ \Upsilon_{41} & 0 & 0 & -R_i & * & * \\ \Upsilon_{51} & 0 & 0 & 0 & \Upsilon_{55} & * \\ \Upsilon_{61} & * & * & * & * & \Upsilon_{66} \end{bmatrix},$$

$$\Phi_{im} = \begin{bmatrix} \Phi_{im}^{11} & * & * & * & * & * & * \\ \Phi_{im}^{21} & \Phi_{im}^{22} & * & * & * & * & * \\ \Phi_{im}^{31} & \Phi_{im}^{32} & \Phi_{im}^{33} & * & * & * & * \\ \Phi_{im}^{41} & \Phi_{im}^{42} & \Phi_{im}^{43} & \Phi_{im}^{44} & * & * & * \\ \Phi_{im}^{51} & \Phi_{im}^{52} & -U_{4i} & U_{4i} & \Phi_{im}^{51} & * & * \\ \tilde{\mathcal{Z}}_{2i}^{\mathrm{T}}P_{im} & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\ \tilde{\mathcal{Z}}_{3i}^{\mathrm{T}}P_{im} & 0 & 0 & 0 & 0 & 0 & -\delta_i J_i \end{bmatrix},$$

$$\Phi_{im}^{11} = sym\{P_{im}\bar{\bar{\mathcal{A}}}_i\} + G_i^{\mathrm{T}}\mathcal{M}_{im}G_i + \tau_M^i G_i^{\mathrm{T}}\mathcal{M}_iG_i + \varepsilon_0^{-1}P_{im} - 4(1+\kappa_i)G_i^{\mathrm{T}}R_iG_i + \sum_{k\in\Lambda_k^m}^{\varrho} \pi_{mk}^i(h)P_{ik}.$$

If $\Upsilon_{im} < 0$, we can get $\mathbb{E}\left\{\mathcal{L}V(\zeta(t),r_t) + \tilde{z}^{\mathrm{T}}(t)\tilde{z}(t) - \gamma^2\tilde{v}^{\mathrm{T}}(t)\tilde{v}(t)\right\} < 0$, the system (14) is stochastically stable with H_∞ performance. Considering that the transition rate is partially unknown, we will draw the appropriate conclusions from two cases as follow:

(1) If $\Lambda_k^m \neq \varnothing$ and $\Lambda_{uk}^m \neq \varnothing$, $m \in \Lambda_k^m$, then the elements $\sum_{k=1}^{\varrho} \pi_{mk}^i(h)P_{ik}$ in Φ_{im}^{11} are equivalent to

$$\sum_{k=1}^{\varrho} \pi_{mk}^i(h)P_{ik} = \left(\sum_{k\in\Lambda_k^m} + \sum_{k\in\Lambda_{uk}^m}\right)\pi_{mk}^i(h)P_{ik} = \sum_{k\in\Lambda_k^m} \pi_{mk}^i(h)P_{ik} - \omega_k^m \sum_{k\in\Lambda_{uk}^m} \frac{\pi_{mk}^i(h)}{-\omega_k^m}P_{ik}, \tag{35}$$

where $\omega_k^m = -\sum_{k\in\Lambda_{uk}^m} \pi_{mk}^i(h)$. Since $\sum_{k\in\Lambda_{uk}} \frac{\pi_{mk}^i(h)}{-\omega_k^m} = 1$, for $\forall l \in \Lambda_{uk}^m$, from (35), we can get

$$\sum_{k=1}^{\varrho} \pi_{mk}^i(h)P_{ik} = \sum_{k\in\Lambda_{uk}^m} \frac{\pi_{mk}^i(h)}{-\omega_k^m}\left(\sum_{k\in\Lambda_k^m} \pi_{mk}^i(h)(P_{ik} - P_{il})\right).$$

According to Schur complement, it is inferred that $\Upsilon_{im} < 0$ is guaranteed by inequality (24), and (25) similarly guarantees $-\mathcal{M}_i + \sum_{k=1}^{\varrho} \pi_{mk}^i(h)\mathcal{M}_{ik} < 0$.

(2) If $\Lambda_k^m \neq \varnothing$ and $\Lambda_{uk}^m \neq \varnothing, m \in \Lambda_{uk}^m$, then the elements $\sum_{k=1}^{\varrho} \pi_{mk}^i(h)P_{ik}$ in Φ_{im}^{11} are equivalent to

$$\begin{aligned}\sum_{k=1}^{\varrho} \pi_{mk}^i(h)P_{ik} &= \sum_{k\in\Lambda_k^m} \pi_{mk}^i(h)P_{ik} + \pi_{mm}^i(h)P_{im} + \sum_{k\in\Lambda_{uk}^m, k\neq m} \pi_{mk}^i(h)P_{ik} \\ &= \sum_{k\in\Lambda_k^m} \pi_{mk}^i(h)P_{ik} + \pi_{mm}^i(h)P_{im} - \rho_k^m \sum_{k\in\Lambda_{uk}^m, k\neq m} \frac{\pi_{mk}^i(h)}{-\rho_k^m}P_{ik},\end{aligned} \tag{36}$$

where $\rho_k^m = -\sum_{k\in\Lambda_{uk}^m} \pi_{mk}^i(h) + \pi_{mm}^i(h)$. Since $\sum_{k\in\Lambda_{uk}^m} \frac{\pi_{mk}^i(h)}{-\rho_k^m} = 1$, for $\forall l \in \Lambda_{uk}^m, k \neq m$, from (36), we can get

$$\sum_{k=1}^{\varrho} \pi_{mk}^i(h)P_{ik} = \sum_{k\in\Lambda_{uk}^m} \frac{\pi_{mk}^i(h)}{-\rho_k^m}\left(\sum_{k\in\Lambda_k^m} \pi_{mk}^i(h)(P_{ik} - P_{il}) + \pi_{mm}^i(h)(P_{im} - P_{il})\right).$$

According to Schur complement, it is inferred that $\Upsilon_{im} < 0$ is guaranteed by inequalities (26) and (27). Meanwhile, for $m \in \Lambda^m_{uk}$, (28) similarly guarantees $-\mathcal{M}_i + \sum_{k=1}^{\rho} \pi^i_{mk}(h)\mathcal{M}_{ik} < 0$.

Theorem 2. *For the given scalars $\bar{u}_{iv_i} > 0, \tau^i_m > 0, \bar{\alpha}_i > 0, \bar{\beta}_i > 0, \varepsilon_i > 0, \varepsilon_0 > 0, \delta_i > 0, \kappa_i \in (0,1)$, the closed-loop system (18) in ellipsoid $\Omega(P_{im}, 1)$ achieves stochastically stable with the guaranteed H_∞ performance for all $\tilde{\omega}_i$, if there exits positive symmetric matrices $X_{im} > 0, \bar{\mathcal{M}}_{im} > 0, \bar{\mathcal{M}}_i > 0, \bar{R}_i > 0, T_i > 0$, and matrices of appropriate dimensions Θ_{im} $\bar{U}_{1i}, \bar{U}_{2i}, \bar{U}_{3i}, \bar{U}_{4i}$ and $\gamma_{ikv_i} = \begin{bmatrix} \gamma^a_{ikv_i} & \gamma^b_{ikv_i} \end{bmatrix}$ with $\gamma^a_{ikv_i}, \gamma^b_{ikv_i} \in R^{1\times n_i}$, the following conditions hold*

$$\begin{bmatrix} \bar{u}_{iv_i} & \gamma_{ikv_i} \\ * & \bar{u}_{iv_i}\bar{P}_i \end{bmatrix} \geqslant 0. \tag{37}$$

(1) If $\Lambda^m_k \neq \emptyset$, and $\Lambda^m_{uk} \neq \emptyset$, $m \in \Lambda^m_k$, for $\forall l \in \Lambda^m_{uk}$, we have

$$\begin{bmatrix} \tilde{\Phi}_{im,\bar{m}} & * & * & * & * & * & * \\ \Upsilon_{21} & \Upsilon_{22} & * & * & * & * & * \\ 0 & \Upsilon_{32} & -I & * & * & * & * \\ \Upsilon_{41} & \Upsilon_{42} & 0 & \Upsilon_{44} & * & & \\ 0 & \Upsilon_{52} & 0 & 0 & \Upsilon_{55} & * & * \\ \Upsilon_{61} & 0 & 0 & 0 & 0 & \Upsilon_{66} & * \\ \Upsilon_{71} & 0 & 0 & 0 & 0 & 0 & \Upsilon_{77} \end{bmatrix} < 0, \tag{38}$$

$$\begin{bmatrix} \tilde{\Phi}_{im,\underline{m}} & * & * & * & * & * & * \\ \Upsilon_{21} & \Upsilon_{22} & * & * & * & * & * \\ 0 & \Upsilon_{32} & -I & * & * & * & * \\ \Upsilon_{41} & \Upsilon_{42} & 0 & \Upsilon_{44} & * & & \\ 0 & \Upsilon_{52} & 0 & 0 & \Upsilon_{55} & * & * \\ \Upsilon_{61} & 0 & 0 & 0 & 0 & \Upsilon_{66} & * \\ \Upsilon_{71} & 0 & 0 & 0 & 0 & 0 & \Upsilon_{77} \end{bmatrix} < 0, \tag{39}$$

$$-\bar{\mathcal{M}}_i + \sum_{k\in\Lambda^m_k} \bar{\pi}^i_{mk}(h)(\bar{\mathcal{M}}_{ik} - \bar{\mathcal{M}}_{il}) < 0, -\bar{\mathcal{M}}_i + \sum_{k\in\Lambda^m_k} \underline{\pi}^i_{mk}(h)(\bar{\mathcal{M}}_{ik} - \bar{\mathcal{M}}_{il}) < 0. \tag{40}$$

where

$$\tilde{\Phi}_{im,\bar{m}} = \begin{bmatrix} \tilde{\Phi}^{11}_{im,\bar{m}} & * & * & * & * & * \\ \tilde{\Phi}^{21}_{im,\bar{m}} & \tilde{\Phi}^{22}_{im,\bar{m}} & * & * & * & * \\ \bar{\Phi}^{31}_{im} & 0 & \bar{\Phi}^{33}_{im} & * & * & * \\ \bar{\Phi}^{41}_{im} & 0 & \bar{\Phi}^{43}_{im} & \bar{\Phi}^{44}_{im} & * & * \\ \bar{\Phi}^{51}_{im} & 0 & \bar{\Phi}^{53}_{im} & \bar{\Phi}^{54}_{im} & \bar{\Phi}^{55}_{im} & * \\ \bar{\Phi}^{61}_{im} & 0 & -U_{4i} & U_{4i} & \bar{\Phi}^{65}_{im} & \bar{\Phi}^{66}_{im} \end{bmatrix}, \tilde{\Phi}_{im,\underline{m}} = \begin{bmatrix} \tilde{\Phi}^{11}_{im,\underline{m}} & * & * & * & * & * \\ \tilde{\Phi}^{21}_{im,\underline{m}} & \tilde{\Phi}^{22}_{im,\underline{m}} & * & * & * & * \\ \bar{\Phi}^{31}_{im} & 0 & \bar{\Phi}^{33}_{im} & * & * & * \\ \bar{\Phi}^{41}_{im} & 0 & \bar{\Phi}^{43}_{im} & \bar{\Phi}^{44}_{im} & * & * \\ \bar{\Phi}^{51}_{im} & 0 & \bar{\Phi}^{53}_{im} & \bar{\Phi}^{54}_{im} & \bar{\Phi}^{55}_{im} & * \\ \bar{\Phi}^{61}_{im} & 0 & -U_{4i} & U_{4i} & \bar{\Phi}^{65}_{im} & \bar{\Phi}^{66}_{im} \end{bmatrix},$$

$$\tilde{\Phi}^{11}_{im,\bar{m}} = sym\Big\{\mathcal{A}_{im}X_{im} + \sum_{v_i=1}^{p_i} \mathcal{B}_{im}\bar{\chi}_{iv_im}T_{v_i}(\Gamma_{v_i}\Theta_{im} + \Gamma^-_{v_i}\gamma^a_{im})\Big\} + \bar{\mathcal{M}}_{im} + \tau^i_M\bar{\mathcal{M}}_i - 4(1+\kappa_i)\bar{R}_i + \sum_{k\in\Lambda_k} \bar{\pi}^i_{mk}(X_{ik} - X_{il}),$$

$$\tilde{\Phi}^{21}_{im,\bar{m}} = \Big[\sum_{v_i=1}^{p_i} \mathcal{B}_{im}\bar{\chi}_{iv_im}T_{v_i}\gamma^b_{im}\Big]^{\mathrm{T}}, \quad \tilde{\Phi}^{21}_{im,\underline{m}} = \Big[\sum_{v_i=1}^{p_i} \mathcal{B}_{im}\bar{\chi}_{iv_im}T_{v_i}\gamma^b_{im}\Big]^{\mathrm{T}}, \quad \tilde{\Phi}^{22}_{im,\bar{m}} = sym\Big\{\mathcal{A}_{im}X_{im}\Big\} + \sum_{k\in\Lambda_k} \bar{\pi}^i_{mk}(X_{ik} - X_{il}),$$

$$\tilde{\Phi}^{11}_{im,\underline{m}} = sym\Big\{\mathcal{A}_{im}X_{im} + \sum_{v_i=1}^{p_i} \mathcal{B}_{im}\bar{\chi}_{iv_im}T_{v_i}(\Gamma_{v_i}\Theta_{im} + \Gamma^-_{v_i}\gamma^a_{im})\Big\} + \bar{\mathcal{M}}_{im} + \tau^i_M\bar{\mathcal{M}}_i - 4(1+\kappa_i)\bar{R}_i + \sum_{k\in\Lambda_k} \bar{\pi}^i_{mk}(X_{ik} - X_{il}),$$

$$\tilde{\Phi}^{22}_{im,\underline{m}} = sym\Big\{\mathcal{A}_{im}X_{im}\Big\} + \sum_{k\in\Lambda_k} \bar{\pi}^i_{mk}(X_{ik} - X_{il}).$$

(2) If $\Lambda^m_k \neq \emptyset$, and $\Lambda^m_{uk} \neq \emptyset$, $m \in \Lambda^m_{uk}$, for $\forall l \in \Lambda^m_{uk}, l \neq m$, we have

$$\begin{bmatrix} \check{\Phi}_{im,\bar{m}} & * & * & * & * & * & * \\ \Upsilon_{21} & \Upsilon_{22} & * & * & * & * & * \\ 0 & \Upsilon_{32} & -I & * & * & * & * \\ \Upsilon_{41} & \Upsilon_{42} & 0 & \Upsilon_{44} & * & & \\ 0 & \Upsilon_{52} & 0 & 0 & \Upsilon_{55} & * & * \\ \Upsilon_{61} & 0 & 0 & 0 & 0 & \Upsilon_{66} & * \\ \Upsilon_{71} & 0 & 0 & 0 & 0 & 0 & \Upsilon_{77} \end{bmatrix} < 0, \tag{41}$$

$$\begin{bmatrix} \check{\Phi}_{im,\underline{m}} & * & * & * & * & * & * \\ \Upsilon_{21} & \Upsilon_{22} & * & * & * & * & * \\ 0 & \Upsilon_{32} & -I & * & * & * & * \\ \Upsilon_{41} & \Upsilon_{42} & 0 & \Upsilon_{44} & * & & \\ 0 & \Upsilon_{52} & 0 & 0 & \Upsilon_{55} & * & * \\ \Upsilon_{61} & 0 & 0 & 0 & 0 & \Upsilon_{66} & * \\ \Upsilon_{71} & 0 & 0 & 0 & 0 & 0 & \Upsilon_{77} \end{bmatrix} < 0, \tag{42}$$

$$X_{im} - X_{il} \geq 0, \quad \bar{\mathcal{M}}_{im} - \bar{\mathcal{M}}_{il} \geq 0, \tag{43}$$

$$-\bar{\mathcal{M}}_i + \sum_{k\in\Lambda_k^m} \bar{\pi}^i_{mk}(h)(\bar{\mathcal{M}}_{ik} - \bar{M}_{il}) < 0, -\bar{\mathcal{M}}_i + \sum_{k\in\Lambda_k^m} \underline{\pi}^i_{mk}(h)(\bar{\mathcal{M}}_{ik} - \bar{\mathcal{M}}_{il}) < 0. \tag{44}$$

where

$$\check{\Phi}_{im,\bar{m}} = \begin{bmatrix} \check{\Phi}^{11}_{im,\bar{m}} & * & * & * & * & * \\ \check{\Phi}^{21}_{im,\bar{m}} & \check{\Phi}^{22}_{im,\bar{m}} & * & * & * & * \\ \bar{\Phi}^{31}_{im} & 0 & \bar{\Phi}^{33}_{im} & * & * & * \\ \bar{\Phi}^{41}_{im} & 0 & \bar{\Phi}^{43}_{im} & \bar{\Phi}^{44}_{im} & * & * \\ \bar{\Phi}^{51}_{im} & 0 & \bar{\Phi}^{53}_{im} & \bar{\Phi}^{54}_{im} & \bar{\Phi}^{55}_{im} & * \\ \bar{\Phi}^{61}_{im} & 0 & -U_{4i} & U_{4i} & \bar{\Phi}^{65}_{im} & \bar{\Phi}^{66}_{im} \end{bmatrix}, \check{\Phi}_{im,\underline{m}} = \begin{bmatrix} \check{\Phi}^{11}_{im,\underline{m}} & * & * & * & * & * \\ \check{\Phi}^{21}_{im,\underline{m}} & \check{\Phi}^{22}_{im,\underline{m}} & * & * & * & * \\ \bar{\Phi}^{31}_{im} & 0 & \bar{\Phi}^{33}_{im} & * & * & * \\ \bar{\Phi}^{41}_{im} & 0 & \bar{\Phi}^{43}_{im} & \bar{\Phi}^{44}_{im} & * & * \\ \bar{\Phi}^{51}_{im} & 0 & \bar{\Phi}^{53}_{im} & \bar{\Phi}^{54}_{im} & \bar{\Phi}^{55}_{im} & * \\ \bar{\Phi}^{61}_{im} & 0 & -U_{4i} & U_{4i} & \bar{\Phi}^{65}_{im} & \bar{\Phi}^{66}_{im} \end{bmatrix},$$

$$\check{\Phi}^{11}_{im,\bar{m}} = sym\Big\{\mathcal{A}_{im}X_{im} + \sum_{v_i=1}^{p_i} \mathcal{B}_{im}\bar{\chi}_{iv_im}T_{v_i}(\Gamma_{v_i}\Theta_{im} + \Gamma^{-}_{v_i}\gamma^a_{im})\Big\} + \bar{\mathcal{M}}_{im} + \tau^i_M\bar{\mathcal{M}}_i - 4(1+\kappa_i)\bar{R}_i + \sum_{k\in\Lambda_k} \bar{\pi}^i_{mk}(X_{ik} - X_{il}),$$

$$\check{\Phi}^{21}_{im,\bar{m}} = \Big[\sum_{v_i=1}^{p_i} \mathcal{B}_{im}\bar{\chi}_{iv_im}T_{v_i}\gamma^b_{im}\Big]^{\mathrm{T}}, \quad \check{\Phi}^{22}_{im,\bar{m}} = sym\Big\{\mathcal{A}_{im}X_{im}\Big\} + \sum_{k\in\Lambda_k} \bar{\pi}^i_{mk}(X_{ik} - X_{il}),$$

$$\check{\Phi}^{11}_{im,\underline{m}} = sym\Big\{\mathcal{A}_{im}X_{im} + \sum_{v_i=1}^{p_i} \mathcal{B}_{im}\bar{\chi}_{iv_im}T_{v_i}(\Gamma_{v_i}\Theta_{im} + \Gamma^{-}_{v_i}\gamma^a_{im})\Big\} + \bar{\mathcal{M}}_{im} + \tau^i_M\bar{\mathcal{M}}_i - 4(1+\kappa_i)\bar{R}_i + \sum_{k\in\Lambda_k} \bar{\pi}^i_{mk}(X_{ik} - X_{il}),$$

$$\check{\Phi}^{21}_{im,\underline{m}} = \Big[\sum_{v_i=1}^{p_i} \mathcal{B}_{im}\bar{\chi}_{iv_im}T_{v_i}\gamma^b_{im}\Big]^{\mathrm{T}}, \quad \check{\Phi}^{22}_{im,\underline{m}} = sym\Big\{\mathcal{A}_{im}X_{im}\Big\} + \sum_{k\in\Lambda_k} \bar{\pi}^i_{mk}(X_{ik} - X_{il}),$$

where $\Theta_{im} = K_{im}X_{im}$. Addtionally, the other matrix elements above are represented accordingly as follows:

$$\begin{aligned}
\bar{\Phi}^{31}_{im} &= -2(1+\kappa_i)\bar{R}_i - \bar{U}_{1i} - \bar{U}_{2i} - \bar{U}_{3i} - \bar{U}_{4i}, \\
\bar{\Phi}^{33}_{im} &= -2(4+\kappa_i)R_i - 2\bar{U}_{1i} - 2\bar{U}_{2i} + 2\bar{U}_{3i} + 2\bar{U}_{4i}, \\
\bar{\Phi}^{41}_{im} &= -\bar{U}_{1i} - \bar{U}_{2i} + \bar{U}_{3i} + \bar{U}_{4i}, \\
\bar{\Phi}^{43}_{im} &= -2(2-\kappa_i)\bar{R}_i + \bar{U}_{1i} - \bar{U}_{2i} - \bar{U}_{3i} + \bar{U}_{4i}, \\
\bar{\Phi}^{44}_{im} &= -\bar{\mathcal{M}}_{im} - 4(2-\kappa_i)\bar{R}_i, \\
\bar{\Phi}^{51}_{im} &= -\bar{U}_{3i}G_i - \bar{U}_{4i}G_i, \\
\bar{\Phi}^{53}_{im} &= \bar{U}_{3i} - \bar{U}_{4i} + 3(2-\kappa_i)\bar{R}_i, \\
\bar{\Phi}^{54} &= 3(2-\kappa_i)\bar{R}_i, \\
\bar{\Phi}^{55}_{im} &= -3(2-\kappa_i)\bar{R}_i, \\
\bar{\Phi}^{61}_{im} &= 3(1+\kappa_i)\bar{R}_iG_i, \\
\bar{\Phi}^{63}_{im} &= -\bar{U}_{2i} + 3(1+\kappa_i)\bar{R}_i, \\
\bar{\Phi}^{66}_{im} &= -3(1+\kappa_i)\bar{R}_i, \\
\Upsilon^{22}_{im} &= diag\{-\gamma^2 I, -\gamma^2 I, -\delta_i J_i, -(N-1)^{-1}P_{im}, -(N-1)^{-1}P_{im}, -I\}, \\
\Upsilon^{32}_{im} &= \begin{bmatrix} \mathcal{D}_{im} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Upsilon^{55}_{im} &= -2\varepsilon_i X_{im} + \varepsilon_i^2\bar{R}_i, \\
\Upsilon^{42}_{im} &= \tau^i_M\begin{bmatrix} \mathcal{G}_{im} & 0 & \mathcal{F}_{ijm} & 0 & 0 & 0 \end{bmatrix}, \\
\Upsilon^{41}_{im} &= \tau^i_M\begin{bmatrix} \mathcal{A}_{im}X_{im} + \mathcal{B}_{im}K_{im} & -\mathcal{B}_{im}K_{im} & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Upsilon^{44}_{im} &= -2\varepsilon_i X_{im} + \varepsilon_i^2\bar{R}_i, \\
\Upsilon^{52}_{im} &= \tau^i_M\begin{bmatrix} s_i(1-\iota_i)\mathcal{G}_{im} & 0 & \mathcal{F}_{ijm} & 0 & 0 & 0 \end{bmatrix}, \\
\Upsilon^{66}_{im} &= diag\{-\varepsilon_0, -\varepsilon_0, -I, -a_ib_iI, -J_i\}, \\
\Upsilon^{77}_{im} &= diag\{-(1-\kappa_i)^{-1}\bar{R}_i, -3(1-\kappa_i)^{-1}\bar{R}_i, -(\kappa_i)^{-1}\bar{R}_i, -3(\kappa_i)^{-1}\bar{R}_i\},
\end{aligned}$$

$$\Upsilon_{im}^{21}=\begin{bmatrix}\mathcal{G}_{im}^{\mathrm{T}} & \mathcal{G}_{im}^{\mathrm{T}} & 0 & 0 & 0 & 0\\ 0 & -\bar{\alpha}_i(1-\bar{\beta}_i)L_{im}^{\mathrm{T}} & 0 & 0 & 0 & 0\\ 0 & -\bar{\alpha}_i\bar{\beta}_i L_{im} & 0 & 0 & 0 & 0\\ \mathcal{F}_{ijm}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0\\ 0 & \mathcal{F}_{ijm}^{\mathrm{T}} & 0 & 0 & 0 & 0\\ \mathcal{E}_{im}X_{im} & 0 & 0 & 0 & 0 & 0\end{bmatrix},\quad \Upsilon_{im}^{61}\begin{bmatrix}X_{im} & 0 & 0 & 0 & 0 & 0\\ 0 & X_{im} & 0 & 0 & 0 & 0\\ -L_{im}^{\mathrm{T}} & L_{im}^{\mathrm{T}} & 0 & 0 & 0 & 0\\ 0 & L_{im}^{\mathrm{T}} & 0 & 0 & 0 & 0\\ 0 & 0 & J_iL_{im}^{\mathrm{T}} & 0 & 0 & 0\end{bmatrix},$$

$$\Upsilon_{im}^{71}=\begin{bmatrix}0 & 0 & \bar{U}_{1i}+\bar{U}_{3i} & -\bar{U}_{1i}+\bar{U}_{3i} & -\bar{U}_{3i} & 0\\ 0 & 0 & \bar{U}_{2i}+\bar{U}_{4i} & -\bar{U}_{2i}+\bar{U}_{4i} & -\bar{U}_{4i} & 0\\ \bar{U}_{1i}+\bar{U}_{3i} & 0 & -\bar{U}_{1i}+\bar{U}_{3i} & 0 & 0 & -\bar{U}_{3i}\\ \bar{U}_{2i}+\bar{U}_{4i} & 0 & -\bar{U}_{2i}+\bar{U}_{4i} & 0 & 0 & -\bar{U}_{4i}\end{bmatrix}.$$

Meanwhile, the corresponding controller and observer gain matrices are given as follows:

$$K_{im}=\Theta_{im}X_{im}^{-1},\quad L_{im}=-X_{im}C_{im}^{\mathrm{T}} \tag{45}$$

Proof. Similar with the method in [19], defining $\gamma_{imv_i}\tilde{P}_{im}^{-1}=\begin{bmatrix}\gamma_{imv_i}^a & \gamma_{imv_i}^b\end{bmatrix}\tilde{P}_{im}^{-1}\triangleq\begin{bmatrix}S_{im}^a & S_{im}^b\end{bmatrix}$, it can be shown that (37) is equivalent to $\Omega(P_{im},1)\subset\Psi(S_{im})$ in (23). Assume $\zeta_i(t)\in\Omega(P_{im},1)$, then define $\tilde{P}_{im}=diag\{P_{im},P_{im}\}$, $\tilde{X}_{im}=\tilde{P}_{im}^{-1}=\begin{bmatrix}P_{im}^{-1} & 0\\ 0 & P_{im}^{-1}\end{bmatrix}\triangleq\begin{bmatrix}X_{im} & 0\\ 0 & X_{im}\end{bmatrix}$, $\bar{R}_i=X_{im}R_iX_{im}$, $\bar{\mathcal{M}}_i=X_{im}\mathcal{M}_iX_{im}$, $\bar{U}_{1i}=X_{im}U_{1i}X_{im}$, $\bar{\mathcal{M}}_{im}=X_{im}\mathcal{M}_{im}X_{im}$, $\bar{U}_{2i}=X_{im}U_{2i}X_{im}$, $\bar{U}_{3i}=X_{im}U_{3i}X_{im}$, $\bar{U}_{4i}=X_{im}U_{4i}X_{im}$, $\Theta_{im}=K_{im}X_{im}$ and $L_{im}=-X_{im}C_{im}^T$. Pre- and post-multiplying the condition (24) and (26) by $diag\{X_{im},X_{im},X_{im},X_{im},X_{im},X_{im},I_{1\times7},R_i^{-1},X_{im},X_{im},X_{im},X_{im}\}$ respectively, and according to $\pi_{mk}^i(h)\in\begin{bmatrix}\underline{\pi}_{mk}^i(h) & \bar{\pi}_{mk}^i(h)\end{bmatrix}$ and Lemma 3, one has the inequalities (38) – (44) hold. Thus, the closed loop system (18) is stochastically stable with H_∞ performance, and the controller and observer gain in (45) can be obtained. This completes the proof. □

4. Illustrative example

In the section, an example will be given to show the effectiveness and applicability of our theoretical findings. We consider a network of 3-machine-9-bus power as a large-scale SMJSs [32]. $q_i^{\mathrm{T}}(t)=[\Delta\delta_i(t)\quad\Delta\omega_i(t)]$ $(i=1,2,3)$ are the states of large-scale power systems with three subsystems, where $\Delta\delta_i(t)$ and $\Delta\omega_i(t)$ denote the deviation of power angle and the deviation of angular velocity. $u_1(t)=\Delta P_{u1}$, $u_i(t)=[\Delta P_{ui}\quad\Delta E_{ui}]$ $(i=2,3)$ are the three subsystem control inputs, where ΔE_{ui} and ΔP_{ui} stand the deviation of excitation voltage and mechanical input power, respectively. The relevant matrix parameters for systems (1) with three switching modes are as follows $(m=1,2,3)$:

$$\mathcal{A}_{11}=\begin{bmatrix}-1 & 376.9911\\ -0.0541 & -0.0212\end{bmatrix},\quad \mathcal{A}_{12}=\begin{bmatrix}-1 & 376.9911\\ -0.0532 & -0.0211\end{bmatrix},\quad \mathcal{A}_{13}=\begin{bmatrix}-1 & 376.9911\\ -0.0538 & -0.0212\end{bmatrix},$$

$$\mathcal{B}_{1m}=\begin{bmatrix}0\\ 0.0212\end{bmatrix},\quad \mathcal{G}_{1m}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix},\quad \mathcal{E}_{1m}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \mathcal{C}_{1m}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad \mathcal{D}_{1m}=\begin{bmatrix}0.1 & 0\\ 0.1 & 0\end{bmatrix},$$

$$\mathcal{F}_{121}=\begin{bmatrix}0 & 0 & 0\\ 0.0338 & 0 & 0.0215\end{bmatrix},\quad \mathcal{F}_{122}=\begin{bmatrix}0 & 0 & 0\\ 0.0338 & 0 & 0.0207\end{bmatrix},\quad \mathcal{F}_{131}=\begin{bmatrix}0 & 0 & 0\\ 0.0202 & 0 & 0.0134\end{bmatrix},$$

$$\mathcal{F}_{132}=\begin{bmatrix}0 & 0 & 0\\ 0.0194 & 0 & 0.0115\end{bmatrix},\quad \mathcal{F}_{123}=\begin{bmatrix}0 & 0 & 0\\ 0.0338 & 0 & 0.0209\end{bmatrix},\quad \mathcal{F}_{133}=\begin{bmatrix}0 & 0 & 0\\ 0.0201 & 0 & 0.0126\end{bmatrix},$$

$$\mathcal{A}_{21}=\begin{bmatrix}-1 & 376.9911 & 0\\ -0.1813 & -0.0781 & -0.2465\\ -0.3278 & 0 & -0.5283\end{bmatrix},\quad \mathcal{A}_{22}=\begin{bmatrix}-1 & 376.9911 & 0\\ -0.1804 & -0.0781 & -0.2454\\ -0.3251 & 0 & -0.5292\end{bmatrix},$$

$$\mathcal{A}_{23}=\begin{bmatrix}-1 & 376.9911 & 0\\ -0.1810 & -0.0781 & -0.2462\\ -0.3262 & 0 & -0.5285\end{bmatrix},\quad \mathcal{A}_{33}=\begin{bmatrix}-1 & 376.9911 & 0\\ -0.3231 & -0.1661 & -0.3912\\ -0.4001 & 0 & -0.6332\end{bmatrix},$$

$$\mathcal{F}_{211}=\begin{bmatrix}0 & 0\\ 0.1302 & 0\\ 0.2200 & 0\end{bmatrix},\quad \mathcal{F}_{212}=\begin{bmatrix}0 & 0\\ 0.1309 & 0\\ 0.2196 & 0\end{bmatrix},\quad \mathcal{F}_{213}=\begin{bmatrix}0 & 0\\ 0.1305 & 0\\ 0.2198 & 0\end{bmatrix},\quad \mathcal{F}_{233}=\begin{bmatrix}0 & 0 & 0\\ 0.0500 & 0 & 0.1068\\ 0.1068 & 0 & 0.1232\end{bmatrix},$$

$$\mathcal{F}_{231}=\begin{bmatrix}0 & 0 & 0\\ 0.0511 & 0 & 0.0309\\ 0.1078 & 0 & 0.1245\end{bmatrix},\quad \mathcal{F}_{232}=\begin{bmatrix}0 & 0 & 0\\ 0.0495 & 0 & 0.0277\\ 0.1056 & 0 & 0.1211\end{bmatrix},\quad \mathcal{B}_{2m}=\begin{bmatrix}0 & 0\\ 0.0781 & 0\\ 0 & 0.1667\end{bmatrix},$$

$$\mathcal{A}_{31}=\begin{bmatrix}-1 & 376.9911 & 0\\ -0.3231 & -0.1661 & -0.3961\\ -0.4036 & 0 & -0.6329\end{bmatrix},\quad \mathcal{A}_{32}=\begin{bmatrix}-1 & 376.9911 & 0\\ -0.3231 & -0.1661 & -0.3890\\ -0.3932 & 0 & -0.6346\end{bmatrix},$$

$$\mathcal{F}_{312}=\begin{bmatrix}0 & 0\\ 0.1893 & 0\\ 0.2321 & 0\end{bmatrix},\quad \mathcal{F}_{311}=\begin{bmatrix}0 & 0\\ 0.1926 & 0\\ 0.2388 & 0\end{bmatrix},\quad \mathcal{F}_{313}=\begin{bmatrix}0 & 0\\ 0.1918 & 0\\ 0.2362 & 0\end{bmatrix},$$

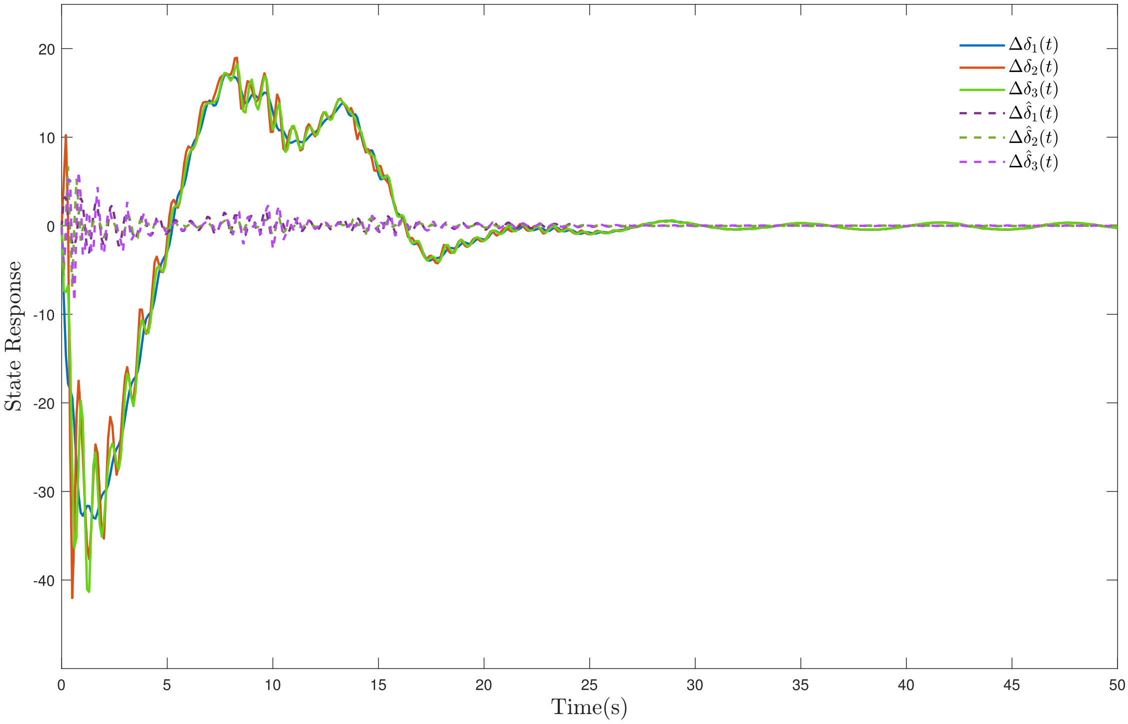

Fig. 2. The state response of angular deviation $\Delta\delta_i(t)$ and estimated state $\Delta\hat{\delta}_i(t)$.

$$\mathcal{F}_{321}=\begin{bmatrix}0&0&0\\0.1304&0&0.1058\\0.1647&0&0.2071\end{bmatrix},\quad \mathcal{F}_{323}=\begin{bmatrix}0&0&0\\0.1298&0&0.1048\\0.1628&0&0.2070\end{bmatrix},\quad \mathcal{F}_{322}=\begin{bmatrix}0&0&0\\0.1278&0&0.1038\\0.1619&0&0.2068\end{bmatrix},$$

$$\mathcal{B}_{3m}=\begin{bmatrix}0&0\\0.1661&0\\0&0.1698\end{bmatrix},\quad \mathcal{G}_{2m}=\mathcal{G}_{3m}=diag\{0.1\quad 0.1\quad 0.1\},$$

$$\mathcal{D}_{2m}=\mathcal{D}_{3m}=\begin{bmatrix}0.1&0&0\\0.1&0&0\\0.1&0&0\end{bmatrix},\quad \mathcal{E}_{2m}=\mathcal{E}_{3m}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\quad \mathcal{C}_{2m}=\mathcal{C}_{3m}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.$$

The transition probability matrix of system (18) is $\begin{bmatrix}\pi^i_{11}(h)&?&\pi^i_{13}(h)\\\pi^i_{21}(h)&?&\pi^i_{23}(h)\\?&\pi^i_{32}(h)&\pi^i_{33}(h)\end{bmatrix}$, where "?" represents a completely unknown transition probability, and $\pi^i_{11}(h)\in[-0.6,-0.3]$, $\pi^i_{13}(h)\in[0.2,0.3]$, $\pi^i_{21}(h)\in[0.4,0.7]$, $\pi^i_{23}(h)\in[0.1,0.2]$, $\pi^i_{32}(h)\in[-0.8,-0.2]$, $\pi^i_{33}(h)\in[0.3,0.4]$. The scalars and positive matrix are chosen as $\delta_i=0.1$, $\epsilon_i=0.1$, $\tau^i_M=0.01$, $\bar{\alpha}_i=0.3$, $\bar{\beta}_i=0.2$ and $\gamma^2=1$, by solving matrix equalities in Theorem 2, we get the weight matrices of the ETM in (4) as follows: $J_1=0.0698, J_2=0.0394, J_3=0.0231$, respectively. The corresponding gain parameters of observer and controller designed are as follows

$$K_{11}=\begin{bmatrix}2.6371&-1.4669\end{bmatrix}, K_{12}=\begin{bmatrix}-2.0597&-0.9901\end{bmatrix},\quad K_{13}=\begin{bmatrix}-6.6846&-0.8588\end{bmatrix},$$

$$L_{11}=\begin{bmatrix}-0.8919&0.0398\\0.0398&-0.0074\end{bmatrix},\quad L_{12}=\begin{bmatrix}-0.5092&0.0233\\0.0233&-0.0063\end{bmatrix},\quad L_{13}=\begin{bmatrix}-0.5494&0.0255\\0.0255&-0.0067\end{bmatrix},$$

$$K_{21}=\begin{bmatrix}-7.6839&-0.3282&-0.2633\\-1.1116&-0.0436&-11.9801\end{bmatrix},\quad K_{22}=\begin{bmatrix}-1.5337&-0.3297&-0.1002\\-0.8636&-0.0078&-6.3123\end{bmatrix},$$

$$K_{23}=\begin{bmatrix}-1.6215&-0.4380&0.4879\\-8.7513&0.0662&-44.6612\end{bmatrix},\quad K_{33}=\begin{bmatrix}1.1442&-0.0468&0.0171\\-5.9818&-0.6298&-118.9027\end{bmatrix},$$

$$L_{21}=\begin{bmatrix}-0.7851&0.0390&0\\0.0390&-0.0104&0\\-0.0152&-0.0008&0\end{bmatrix},\quad L_{22}=\begin{bmatrix}-0.5753&0.0298&0\\0.0298&-0.0077&0\\-0.0078&-0.0006&0\end{bmatrix},\quad L_{23}=\begin{bmatrix}-0.8397&0.0335&0\\0.0335&-0.0081&0\\-0.0249&-0.0033&0\end{bmatrix},$$

$$K_{31}=\begin{bmatrix}-0.1314&-0.2324&-0.0847\\-3.5529&0.015&-11.2343\end{bmatrix},\quad K_{32}=\begin{bmatrix}-0.5595&-0.1237&-0.0405\\-1.0940&-0.0129&-5.7918\end{bmatrix},$$

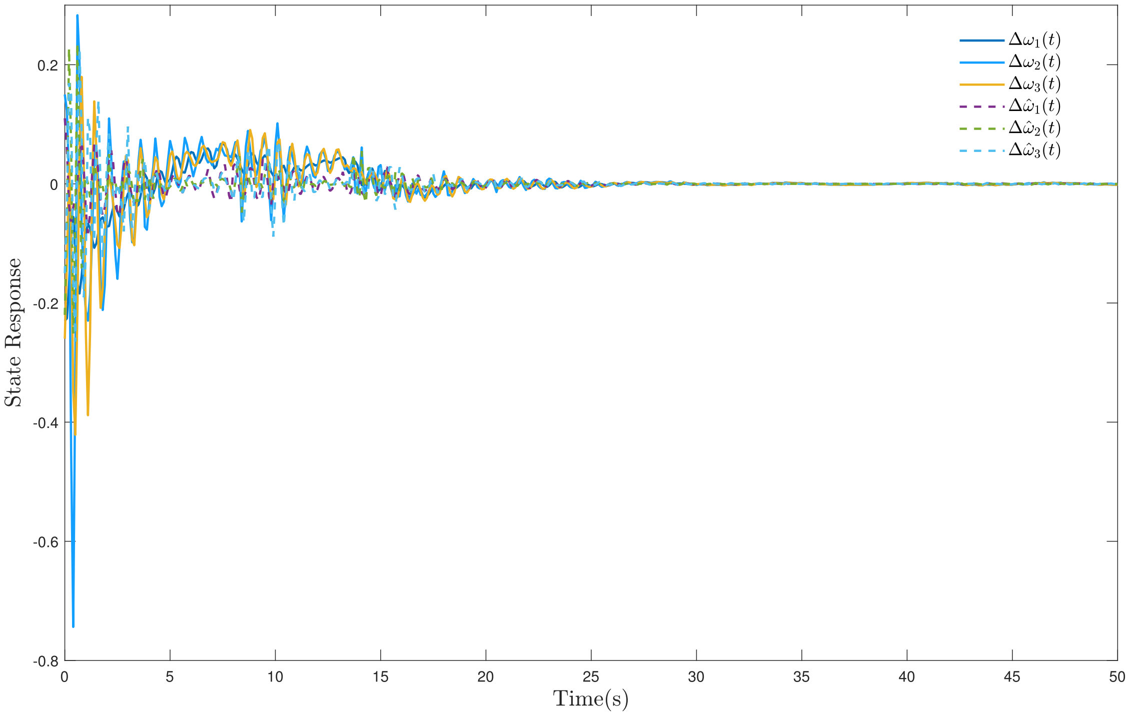

Fig. 3. The angular velocity deviation $\Delta\omega_i(t)$ and estimated state $\Delta\hat{\omega}_i(t)$.

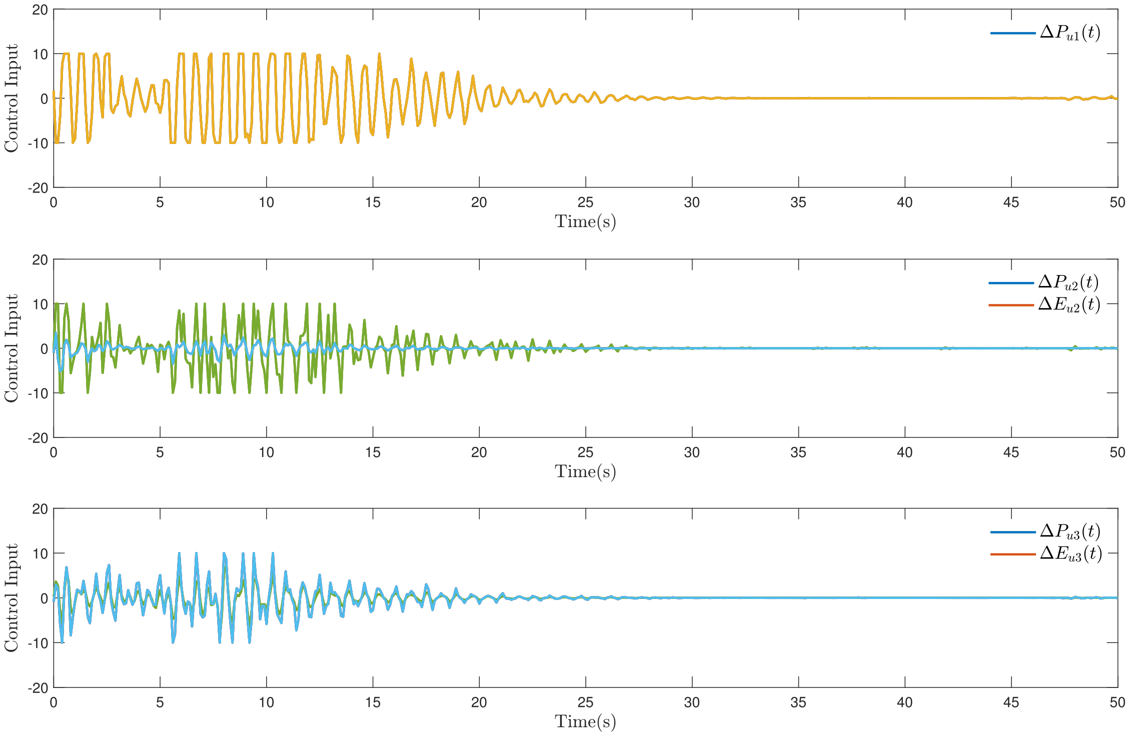

Fig. 4. The control input of the systems with saturation level $\bar{u}_i = 10$.

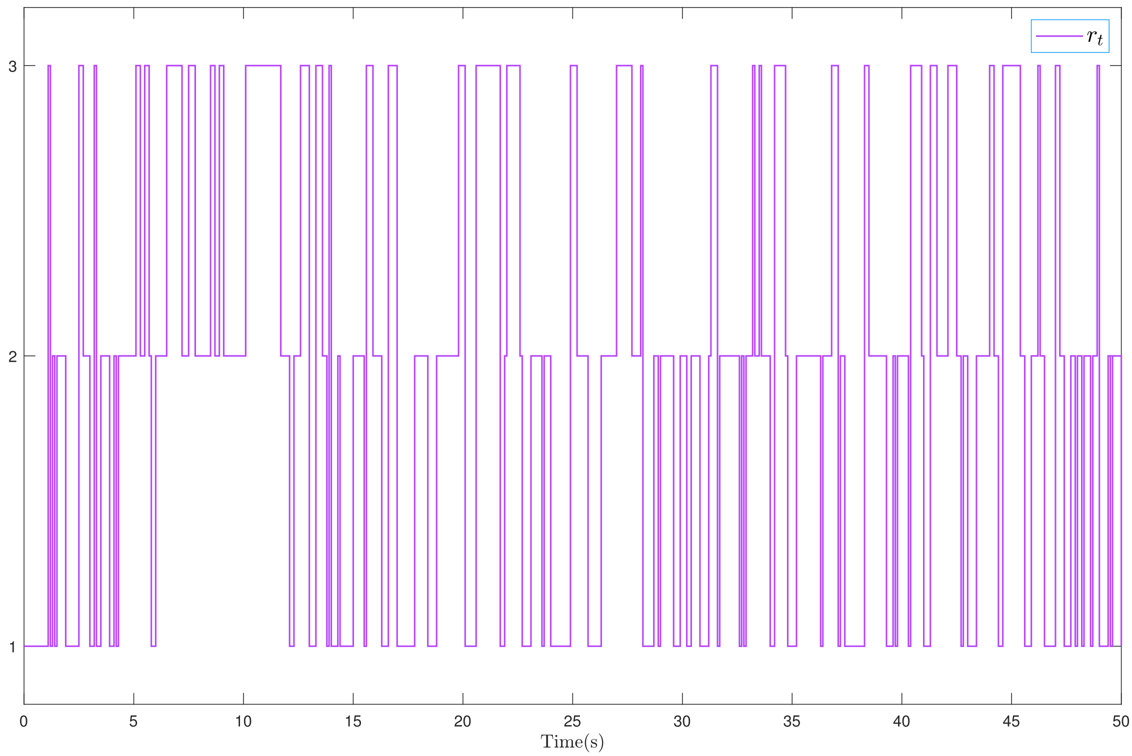

Fig. 5. The switching state of semi-Markovian process.

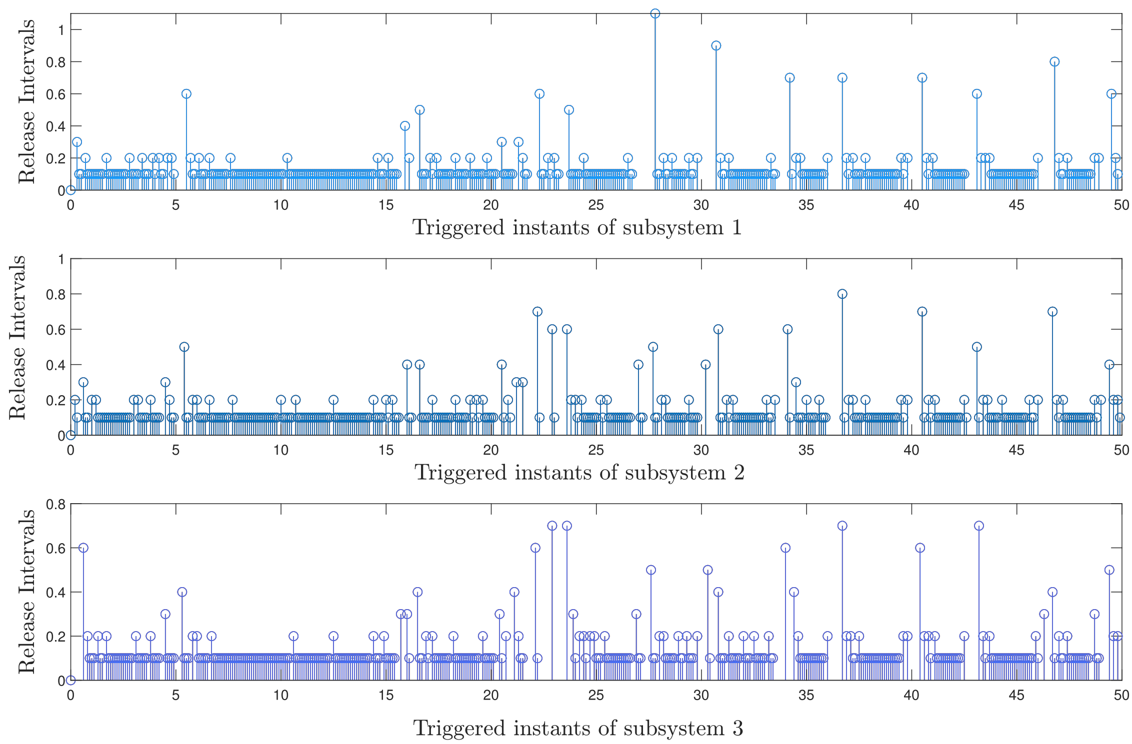

Fig. 6. The released instants and intervals of the three subsystems.

$$L_{31} = \begin{bmatrix} -1.0697 & 0.0469 & 0 \\ 0.0469 & -0.0084 & 0 \\ -0.0097 & -0.0020 & 0 \end{bmatrix}, \quad L_{32} = \begin{bmatrix} -0.5448 & 0.0243 & 0 \\ 0.0243 & -0.0066 & 0 \\ -0.0129 & -0.0009 & 0 \end{bmatrix}, \quad L_{33} = \begin{bmatrix} -0.4420 & 0.0171 & 0 \\ 0.0171 & -0.0051 & 0 \\ -0.0145 & -0.0008 & 0 \end{bmatrix}.$$

The initial conditions are given as $q_1(0) = \begin{bmatrix} 0.13 & -0.23 \end{bmatrix}^{\mathrm{T}}$, $q_2(0) = \begin{bmatrix} -0.16 & 0.15 & -0.21 \end{bmatrix}^{\mathrm{T}}$, $q_3(0) = \begin{bmatrix} 0.11 & -0.22 & 0.18 \end{bmatrix}^{\mathrm{T}}$. The initial values of the observer state are respectively: $\hat{q}_1(0) = \begin{bmatrix} -0.25 & 0.11 \end{bmatrix}^{\mathrm{T}}$, $\hat{q}_2(0) = \begin{bmatrix} 0.12 & -0.22 & 0.18 \end{bmatrix}^{\mathrm{T}}$, $\hat{q}_3(0) = \begin{bmatrix} -0.22 & -0.15 & 0.21 \end{bmatrix}^{\mathrm{T}}$, the initial value of the dynamic parameter in the adaptive event triggering mechanism is $\varepsilon_i(0) = 0.1$, and the external disturbance of system is $v_i(t) = 0.1sin(t)$, network attack function is $f_i(t) = 0.1tanh(0.1t)$. Figs. 2 and 3 depict the angular deviation $\Delta\delta_i(t)$, angular velocity deviation $\Delta\omega_i(t)$ and estimated state $\Delta\hat{\delta}_i(t)$, $\Delta\hat{\omega}_i(t)$ of the subsystem. Fig. 4 shows the control input of the systems. Despite the simultaneous presence of multiple network attacks and actuator saturation, the system trajectory converges within approximately 30 s. This demonstrates that the proposed control strategy exhibits pronounced robustness, effectively mitigating the influence of cyberattacks while preserving system stability. Figs. 5 and 6 display, respectively, the switching states of the semi-Markovian process and the data-release instants and intervals of the subsystem. Importantly, once convergence is achieved, the activation frequency of the dynamic memory event-triggering mechanism decreases substantially, thereby yielding significant energy savings.

5. Conclusion

This paper has studied the decentralized stabilization method for a class of large-scale SMJSs with actuator saturation and network attackers. To begin with, the adaptive event-tiggering mechanism improved the utilization efficiency of communication resources, and the random occuring hybid attacks in the communication channel was considered. Then the novel observer-based control model for the considered large-scale systems have been constructed. By applying the theory of mode-depended Lyapunov functions and LMI techniques, we obtain some appropriate criteria to ensure stochastic stabilization of the system under consideration with H_∞ performance. Then, the LMIs method can be utilized to simultaneously obtain the weighting parameter of trigger, and the controller and observer gains. Finally, we simulated an example of power systems and confirmed the effectiveness of our proposed theoretical findings and methods. In addition, future research will focus on extending data-driven control method [51] to large-scale SMJSs under adaptive event-triggered mechanisms.

Data availability

The authors do not have permission to share data.

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