

Distributed Formation Control for Second-Order Nonlinear Multiagent Systems Using Predictor-Based Accelerated Fuzzy Learning

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Abstract—This article investigates the distributed formation control problem of second-order multiagent systems (MASs) subject to unknown nonlinear dynamics. A predictor-based fuzzy learning framework is developed, in which fuzzy logic systems approximate the unknown nonlinear dynamics and a two-layer learning mechanism accelerates weight adaptation by utilizing the velocity prediction error. A distributed observer is designed for jointly connected switching directed topologies to handle limited leader accessibility, allowing agents to estimate the global reference trajectory using solely local information. Furthermore, an actor-critic reinforcement learning architecture is integrated to realize distributed optimal formation control, in which the value function is reformulated with a compensatory term that exploits available system structural information, thereby enhancing policy optimization efficiency and accelerating convergence. Simulation results demonstrate that the proposed approach exhibits improved robustness and accuracy in both dynamics approximation and position tracking when compared with conventional learning-based methods.

Index Terms—Formation control, fuzzy logic systems (FLSs), multiagent systems (MASs), reinforcement learning.

I. INTRODUCTION

SECOND-ORDER multiagent systems (MASs) have emerged as a fundamental research paradigm in cooperative control, aiming to achieve coordinated behaviors among multiple interacting dynamical agents governed by nonlinear dynamics [1]. Owing to their inherent position-velocity coupling, second-order MASs provide an effective and realistic framework for modeling a wide range of physical systems with inertial characteristics. Typical coordination objectives include

consensus [2], synchronization, formation maintenance [3], flocking, and cooperative tracking, which play a pivotal role in practical applications such as uncrewed aerial vehicles, uncrewed surface vessels (USVs), autonomous vehicle platoons, satellite clusters, robotic networks, and distributed energy systems. Compared with linear or first-order models, second-order nonlinear MASs offer a more faithful representation of real-world systems [4]. However, this increased modeling fidelity inevitably brings substantial challenges, including strong nonlinearities, unmodeled dynamics, parameter uncertainties, and external disturbances [5]. These issues significantly complicate controller synthesis, particularly in distributed settings where only local information exchange is available, thereby strongly motivating the development of distributed, robust, and adaptive formation control strategies for second-order nonlinear MASs.

Among various cooperative control problems, formation control has attracted particular attention [6], [7], which enables groups of agents to achieve and maintain prescribed geometric configurations while performing complex tasks [8], [9]. Formation control not only enhances mission efficiency and coverage but also improves safety and robustness by enabling distributed task execution [10]. In the past decade, a considerable amount of research has been devoted to the formation control of MASs, and a variety of methodologies have been proposed. In [11], an adaptive output-feedback formation control method is proposed for stochastic nonlinear MASs, ensuring unknown dynamics identification and interagent collision avoidance. In [12], an event-triggered predefined-time formation control method is proposed for second-order MASs with reduced communication. A stochastic feedback-based formation control approach for second-order MASs is proposed in [13], enabling smooth trajectories and reduced actuator load. An active disturbance rejection formation control method for MASs subject to input constraints is proposed in [14], combining an intermediate estimator and an improved model-predictive controller. More recently, learning-based methods [15], [16] have been introduced, leveraging data-driven modeling and approximation capabilities to enhance adaptability in dynamic environments. Despite these advances, many existing results still rely on restrictive assumptions, such as Lipschitz continuity of nonlinear dynamics and availability of precise system models, which may not hold in practical MASs [17]. These theoretical constraints significantly limit the capacity of the system to cope with inherent

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information from agent j at time t , and $a_{ij}(\eta) = 0$ otherwise. The Laplacian matrix associated with $\mathcal{G}(\eta)$ is defined as $\mathcal{L}(\eta) = \mathcal{D}(\eta) - \mathcal{A}(\eta)$, where $\mathcal{D}(\eta) = \text{diag}\{d_1(\eta), \dots, d_N(\eta)\}$ with $d_i(\eta) = \sum_{j=1}^N a_{ij}(\eta)$. To characterize intermittent communication, jointly connected switching topologies are considered, where the union graph over each interval $[t_k, t_{k+1})$ is defined as $\mathcal{G}([t_k, t_{k+1})) = (\mathcal{V}, \cup_{t \in [t_k, t_{k+1})) \mathcal{E}(\eta))$.

Assumption 1: The switching communication topology $\mathcal{G}(\eta)$ is jointly connected; that is, for every interval $[t_k, t_{k+1})$, the union graph $\mathcal{G}([t_k, t_{k+1}))$ contains a spanning tree with the leader as its root.

B. System Model

A class of second-order nonlinear MASs is considered, where each agent is described by position dynamics and nonlinear velocity dynamics subject to external disturbances. The dynamics of agent i are modeled as

$$\begin{cases} \dot{x}_i(t) = A_i(x_i)v_i(t) \\ \dot{v}_i(t) = f_i(v_i(t)) + B_i(x_i)u_i(t) + \omega_i(t) \end{cases} \quad (1)$$

where $x_i(t) \in \mathbb{R}^n$ denotes the position of agent i , $v_i(t) \in \mathbb{R}^n$ represents the velocity vector, and $u_i(t) \in \mathbb{R}^n$ is the control input. $A_i(x_i) \in \mathbb{R}^{n \times n}$ and $B_i(x_i) \in \mathbb{R}^{n \times n}$ are the system matrices that facilitate coordinate transformations between different reference frames. $f_i(v_i(t)) \in \mathbb{R}^n$ denotes the intrinsic nonlinear velocity dynamics and unmodeled uncertainties, and $\omega_i(t) \in \mathbb{R}^n$ is the external environmental disturbances.

Remark 1: The system matrices $A_i(x_i)$ and $B_i(x_i)$ in (1) are employed to map velocities and control inputs between different reference frames, such as body-fixed and inertial frames. In particular, if $A_i(x_i) = I_n$ and $B_i(x_i) = I_n$, the dynamics (1) reduces to the conventional second-order MASs models studied in the papers [3], [8], [32]. By suitably selecting $A_i(x_i)$ and $B_i(x_i)$, this dynamics (1) can readily accommodate a wide range of practical second-order MASs, including USVs and other vehicles with orientation dependent dynamics. Compared with standard second-order agent models, this formulation offers enhanced modeling flexibility.

C. Control Objective

The reference trajectory $r(t) \in \mathbb{R}^n$ is generated by $\dot{r}(t) = f_0(r(t))$. Each agent i is required to asymptotically track the reference trajectory $x_0(t)$ and maintain a prescribed formation pattern, given by

$$\lim_{t \rightarrow \infty} \|x_i(t) - r(t) - d_i\| \leq \varpi_i, \quad i = 1, \dots, N \quad (2)$$

where $d_i \in \mathbb{R}^n$ denotes the desired formation displacement of agent i relative to the reference trajectory, and $\varpi_i \in \mathbb{R}^+$ represents a bounded error tolerance.

Definition 1: Consider the nonlinear system $\dot{z}(t) = g(z(t))$ with $z(t) \in \mathbb{R}^n$. The system state $z(t)$ is said to be uniformly ultimately bounded (UUB) if there exist constants $\kappa > 0$ and $T > 0$, independent of the initial condition $z(0)$, such that $\|z(t)\| \leq \kappa$ for all $t \geq T$.

III. MAIN RESULTS

To provide a clear overview of the proposed approach, the structure of the distributed formation control scheme is illustrated in Fig. 1. The proposed framework combines a predictor-based fuzzy learning approach for approximating unknown dynamics, a distributed reference trajectory observer via local neighbor communication, and an actor-critic reinforcement learning scheme for distributed optimal formation control. The details of each component are presented in the following sections.

A. Predictor-Based Fuzzy Learning Approach

To address the unknown nonlinear dynamics $f_i(v_i)$, the FLS is adopted as a universal approximator [33]. The membership function of the j th fuzzy rule with respect to the l th element of v_i is defined as

$$\mu_{ij}(v_{il}) = \exp\left(-\frac{(v_{il} - h_{ij})^2}{2w_{ij}^2}\right) \quad (3)$$

where $h_{ij} \in \mathbb{R}$ and $w_{ij} \in \mathbb{R}^+$ denote the center and width of the Gaussian membership function, respectively.

Based on the fuzzy inference mechanism, the nonlinear function $f_i(v_i)$ can be approximated as

$$f_i(v_i(t)) = \frac{\sum_{j=1}^m \Gamma_{ij} \prod_{l=1}^n \mu_{ij}(v_{il})}{\sum_{j=1}^m \prod_{l=1}^n \mu_{ij}(v_{il})} + \epsilon_i(t) \quad (4)$$

where Γ_{ij} is the adjustable parameter vector associated with the j th rule, m is the number of fuzzy rules, and $\epsilon_i(t)$ denotes the approximation error.

By introducing the fuzzy basis function

$$\psi_{i,\ell}(v_i) = \frac{\prod_{l=1}^n \mu_{i,\ell}(v_{il})}{\sum_{j=1}^m \prod_{l=1}^n \mu_{ij}(v_{il})}, \quad \ell = 1, 2, \dots, m \quad (5)$$

the dynamics (4) can be expressed in a compact form as

$$f_i(v_i) = \Gamma_i^T \psi_i(v_i) + \epsilon_i(t) \quad (6)$$

where $\Gamma_i = [\Gamma_{i1}^T, \Gamma_{i2}^T, \dots, \Gamma_{im}^T]^T \in \mathbb{R}^{m \times n}$ is the ideal FLS weight and $\psi_i(v_i) = [\psi_{i1}(v_i), \psi_{i2}(v_i), \dots, \psi_{im}(v_i)]^T \in \mathbb{R}^m$ is the fuzzy basis function vector.

Accordingly, the FLS-based estimate of the unknown nonlinear dynamics is given by

$$\hat{f}_i(v_i) = \hat{\Gamma}_i^T(t) \psi_i(v_i) \quad (7)$$

where $\hat{\Gamma}_i(t)$ denotes the estimate of the ideal FLS weight Γ_i .

To improve the convergence speed and approximation accuracy of FLS-based estimate (7), a predictor-based accelerated learning scheme is proposed for the adaptive online tuning of the FLS weight parameters $\hat{\Gamma}_i(t)$. The predictor of the velocity vector v_i is described by

$$\hat{v}_i(t) = \hat{\Gamma}_i^T(t) \psi_i(v_i) + B_i(x_i)u_i(t) - K_{i1} \tilde{v}_i(t) \quad (8)$$

where $\tilde{v}_i(t) = \hat{v}_i(t) - v_i(t)$ represents the velocity prediction error and $K_{i1} \in \mathbb{R}^{n \times n}$ is a positive definite gain matrix.

The dynamics of the prediction error can be expressed as

$$\dot{\tilde{v}}_i(t) = \tilde{\Gamma}_i^T(t) \psi_i(v_i) - K_{i1} \tilde{v}_i(t) - \epsilon_i(t) \quad (9)$$

where $\tilde{\Gamma}_i(t) = \hat{\Gamma}_i(t) - \Gamma_i(t)$ is the weight estimation error.

To adaptively update the FLS weights, a two-layer predictor-based learning law is given by

$$\dot{\hat{\Gamma}}_i(t) = -c_{i1} \left(\hat{\Gamma}_i(t) - \Theta_i(t) \right) \quad (10)$$

$$\dot{\Theta}_i(t) = -c_{i2} \frac{\psi_i(v_i)}{1 + \psi_i^T(v_i)\psi_i(v_i)} \theta_i^T(t) \quad (11)$$

where $\theta_i(t) = \dot{\tilde{v}}_i(t) + K_{i1}\tilde{v}_i(t)$ and $c_{i1}, c_{i2} \in \mathbb{R}^+$ are the design learning rates.

Assumption 2: The environmental disturbances and the approximation errors are bounded. There exist positive constants $\bar{\omega}_i$ and $\bar{\epsilon}_i$ such that the inequalities $\|\omega_i(t)\| \leq \bar{\omega}_i$ and $\|\epsilon_i(t)\| \leq \bar{\epsilon}_i$ hold for all $t \geq 0$ and $i = 1, \dots, N$.

B. Reference Trajectory Observer Design

Given the switching communication topology $\mathcal{G}(\eta)$, agents are subject to intermittent access to the leader or their neighbors. To address this challenge, a distributed observer is designed to simultaneously estimate the reference trajectory and learn its unknown dynamics, which are modeled as

$$f_0(r(t)) = S^T \psi_0(r) + \vartheta(t) \quad (12)$$

where $S \in \mathbb{R}^{m \times n}$ is the ideal FLS weight, $\psi_0(r) \in \mathbb{R}^m$ is the fuzzy basis function vector, and $\vartheta(t)$ denotes the bounded approximation error satisfying $\|\vartheta(t)\| \leq \bar{\vartheta}$.

Since the true reference trajectory $r(t)$ is unavailable to the followers, the observer constructs the approximate dynamics using the estimated weight matrix $\hat{S}_i(t)$

$$\hat{f}_0(\hat{r}_i(t)) = \hat{S}_i^T(t) \psi_0(\hat{r}_i). \quad (13)$$

To achieve consensus on the reference trajectory under the jointly connected switching topology $\mathcal{G}(\eta)$, the composite observer error $\tilde{r}_i(t)$ is defined as

$$\tilde{r}_i(t) = \sum_{j=1}^N a_{ij}(\eta) (\hat{r}_i(t) - \hat{r}_j(t)) + \beta_i(\eta) (\hat{r}_i(t) - r(t)) \quad (14)$$

where $\beta_i(\eta) > 0$ indicates that agent i has direct access to the leader at time t . Then, the distributed reference trajectory observer is designed as

$$\dot{\hat{r}}_i(t) = \hat{S}_i^T(t) \psi_0(\hat{r}_i) - K_{i2}(t) \tilde{r}_i(t) \quad (15)$$

where $K_{i2}(t) = \varrho_{i1} + \varrho_{i2}e^{-\varrho_{i3}t}$ is the observer gain with $\varrho_{i1}, \varrho_{i2}, \varrho_{i3} \in \mathbb{R}^+$.

To capture the true leader dynamics, the weight matrix $\hat{S}_i(t)$ is updated via a normalized gradient adaptation law

$$\dot{\hat{S}}_i(t) = -c_{i3} \hat{S}_i(t) - c_{i4} \tilde{\psi}_0(\hat{r}_i) \tilde{r}_i^T(t) \quad (16)$$

where $\tilde{\psi}_0(\hat{r}_i) = \psi_0(\hat{r}_i)/(1 + \psi_0^T(\hat{r}_i)\psi_0(\hat{r}_i))$ and $c_{i3}, c_{i4} \in \mathbb{R}^+$ are the design parameters.

Let $\delta_i(t) = \hat{r}_i(t) - r(t)$ be the local observer error of agent i relative to the true reference trajectory. The global error vector is denoted as $\tilde{r} = [\tilde{r}_1^T, \dots, \tilde{r}_N^T]^T$ and $\delta = [\delta_1^T, \dots, \delta_N^T]^T$. The

observer error (14) can be written as

$$\tilde{r}(t) = (\mathcal{H}(\eta) \otimes I_n) \delta(t) \quad (17)$$

where $\mathcal{H}(\eta) = \mathcal{L}(\eta) + \mathcal{B}(\eta)$ is positive definite according to Assumption 1. The time derivative of $\delta_i(t)$ is given by

$$\dot{\delta}_i(t) = \hat{S}_i^T(t) \psi_0(\hat{r}_i) - K_{i2}(t) \tilde{r}_i(t) - S^T \psi_0(r) - \vartheta(t). \quad (18)$$

Adding and subtracting $S^T \psi_0(\hat{r}_i)$ to (18) yields

$$\dot{\delta}_i(t) = \tilde{S}_i^T(t) \psi_0(\hat{r}_i) - K_{i2}(t) \tilde{r}_i(t) + \Omega_i(t) \quad (19)$$

where $\Omega_i(t) = S^T(\psi_0(\hat{r}_i) - \psi_0(r)) - \vartheta(t)$.

Assumption 3: The residual error of the reference trajectory observer is bounded. There exists a positive constant $\bar{\Omega}_i$ such that the inequality $\|\Omega_i(t)\| \leq \bar{\Omega}_i$ holds for all $t \geq 0$.

C. Distributed Formation Controller Design

Based on the estimated reference trajectory $\hat{r}_i(t)$, the position tracking error of agent i is defined as

$$\rho_i(t) = x_i(t) - \hat{r}_i(t) - d_i. \quad (20)$$

To achieve formation tracking, a desired velocity command $v_i^d(t)$ is designed based on (20) as

$$v_i^d(t) = A_i^{-1}(x_i) \left(-K_{i3} \rho_i(t) + \dot{\hat{r}}_i(t) \right) \quad (21)$$

where $K_{i3} \in \mathbb{R}^{n \times n}$ is a positive definite gain matrix.

In order to design an optimal controller based on the desired velocity command $v_i^d(t)$ in (21), the velocity tracking error of agent i is defined as

$$e_i(t) = v_i(t) - v_i^d(t). \quad (22)$$

To simultaneously ensure error convergence and reasonable control cost, the following performance index is introduced:

$$J_i(e_i, u_i) = \int_0^\infty U_i(e_i, u_i) ds \quad (23)$$

where $U_i(e_i, u_i) = e_i^T(t) P_i e_i(t) + u_i^T(t) Q_i u_i(t)$ is the instantaneous cost function, with $P_i \in \mathbb{R}^{n \times n}$ and $Q_i \in \mathbb{R}^{n \times n}$ being symmetric positive definite weighting matrices.

The associated value function is given by

$$V_i(e_i) = \int_t^\infty U_i(e_i, u_i) ds \quad (24)$$

and its optimal form is expressed as

$$V_i^*(e_i) = \min_{u_i \in \Pi_{u_i}} \int_t^\infty U_i(e_i, u_i) ds \quad (25)$$

where Π_{u_i} denotes the admissible control set.

By optimal control theory [27], the optimal value function $V_i^*(e_i)$ satisfies the Hamilton–Jacobi–Bellman equation

$$H_i(e_i, u_i) = U_i(e_i, u_i) + (V_{e_i}^*(e_i))^T (f_i(v_i) + B_i(x_i)u_i(t) - \dot{v}_i^d(t) + \omega_i(t)) \quad (26)$$

where $V_{e_i}^*(e_i) = \partial V_i^*(e_i) / \partial e_i$. Setting $\partial H_i(e_i, u_i) / \partial u_i = 0$, the corresponding optimal control law can be derived as

$$u_i^*(e_i) = -\frac{1}{2} Q_i^{-1} B_i^T(x_i) V_{e_i}^*(e_i). \quad (27)$$

To accelerate the learning of optimal control policies and enhance the robustness, the value function is reformulated by incorporating available system structural information as

$$V_i^*(e_i) = \tilde{V}_i(e_i) + \Psi_i(t) \quad (28)$$

where $\tilde{V}_i(e_i) = V_i^*(e_i) - \Psi_i(t)$ with $\Psi_i(t) \in \mathbb{R}$ being a compensatory term to be designed.

Since the optimal value function is generally difficult to obtain analytically, it is common to approximate it using radial basis function neural networks [34]. Accordingly, the optimal value function (28) can be represented by

$$V_i^*(e_i) = W_i^T \phi_i(e_i) + \Psi_i(t) + \varsigma_i(t) \quad (29)$$

where $W_i \in \mathbb{R}^q$ is the ideal neural networks weight, $\phi_i(e_i) \in \mathbb{R}^q$ denotes the radial basis functions, and $\varsigma_i(t) \in \mathbb{R}$ is the approximation error.

An online-updated critic network is employed to learn the ideal neural network weights W_i , formulated as

$$\hat{V}_i(e_i) = \hat{W}_{ic}^T(t) \phi_i(e_i) + \Psi_i(t) \quad (30)$$

where $\hat{W}_{ic}(t) \in \mathbb{R}^q$ denotes the critic network weight. The gradient of $\hat{V}_i(e_i)$ with respect to e_i is given by

$$\hat{V}_{e_i}(e_i) = \phi_{e_i}^T(e_i) \hat{W}_{ic}(t) + \Psi_{e_i}(e_i) \quad (31)$$

with $\phi_{e_i}(e_i) = \partial \phi_i(e_i) / \partial e_i(t)$ and $\Psi_{e_i}(e_i) = \partial \Psi_i(t) / \partial e_i(t)$.

Based on (27) and (31), the corresponding optimal control policy is expressed as

$$\hat{u}_i(e_i) = -\frac{1}{2} Q_i^{-1} B_i^T(x_i) \left(\phi_{e_i}^T(e_i) \hat{W}_{ic}(t) + \Psi_{e_i}(e_i) \right). \quad (32)$$

To facilitate real-time approximation of the optimal value function, the critic network weight $\hat{W}_{ic}(t)$ is updated by

$$\dot{\hat{W}}_{ic}(t) = -b_{i1} \phi_i(e_i) \phi_i^T(e_i) \hat{W}_{ic}(t) \quad (33)$$

where $b_{i1} \in \mathbb{R}^+$ denotes the learning rate of the critic network.

Motivated by the command velocity tracking error $e_i(t)$ in (22), the position tracking error $\rho_i(t)$ in (20), and the FLS-based estimate of nonlinear dynamics $\hat{f}_i(v_i)$ in (7), the compensatory term $\Psi_i(t)$ is constructed as

$$\begin{aligned} \Psi_i(t) = & \left(K_{i3} e_i(t) + 2K_{i4} \rho_i(t) + 2\hat{\Gamma}_i^T(t) \psi_i(v_i) \right)^T \\ & \times (B_i^{-1}(x_i))^T Q_i B_i^{-1}(x_i) e_i(t) \end{aligned} \quad (34)$$

where $K_{i3} = K_{i3}^T \in \mathbb{R}^{n \times n}$ and $K_{i4} \in \mathbb{R}^{n \times n}$ are positive definite gain matrices.

The derivative of the compensatory term $\Psi_i(t)$ with respect to the tracking error e_i , $\Psi_{e_i}(e_i)$ is further expanded as

$$\begin{aligned} \Psi_{e_i}(e_i) = & 2(B_i^{-1}(x_i))^T Q_i B_i^{-1}(x_i) \left(K_{i3} e_i(t) \right. \\ & \left. + K_{i4} \rho_i(t) + \hat{\Gamma}_i^T(t) \psi_i(v_i) \right). \end{aligned} \quad (35)$$

By substituting (35) into (32), the final actor network-based control law is derived as

$$\hat{u}_i(e_i) = -\frac{1}{2} Q_i^{-1} B_i^T(x_i) \phi_{e_i}^T(e_i) \hat{W}_{ia}(t) - B_i^{-1}(K_{i3} e_i(t)$$

$$+ K_{i4} \rho_i(t) + \hat{\Gamma}_i^T(t) \psi_i(v_i)) \quad (36)$$

where $\hat{W}_{ia}(t) \in \mathbb{R}^q$ denotes the actor network weight.

To achieve online adaptation, the actor network weight $\hat{W}_{ia}(t)$ in (36) is updated by

$$\begin{aligned} \dot{\hat{W}}_{ia}(t) = & -\phi_i(e_i) \phi_i^T(e_i) \left(b_{i2} \hat{W}_{ic}(t) \right. \\ & \left. + b_{i3} \left(\hat{W}_{ia}(t) - \hat{W}_{ic}(t) \right) \right) \end{aligned} \quad (37)$$

where $b_{i2}, b_{i3} \in \mathbb{R}^+$ denote the learning rates.

D. Stability Analysis

Theorem 1: Consider the FLS-based estimate (7) implemented with the velocity predictor (8). The prediction error $\tilde{v}_i$ and weight estimation error $\tilde{\Gamma}_i$ are guaranteed to be uniformly bounded, provided that the gain matrix K_{i1} and the learning rates c_{i1}, c_{i2} are selected to satisfy $c_{i1} \geq 2c_{i2}$ and $2\lambda_{\min}(K_{i1})\bar{\psi}_i(v_i) \geq 1$ where $\bar{\psi}_i(v_i) = 1/(1 + \psi_i^T(v_i)\psi_i(v_i))$.

Proof: Construct the Lyapunov candidate function as

$$L_{iv} = \frac{1}{2} \tilde{v}_i^T \tilde{v}_i + \frac{1}{c_{i2}} \|\Theta_i - \Gamma_i\|_F^2 + \frac{1}{c_{i2}} \|\hat{\Gamma}_i - \Theta_i\|_F^2. \quad (38)$$

Taking the time derivative of L_{iv} yields

$$\begin{aligned} \dot{L}_{iv} = & \tilde{v}_i^T \left(\tilde{\Gamma}_i^T \psi_i(v_i) - K_{i1} \tilde{v}_i - \epsilon_i \right) - 2 \operatorname{tr} \left((\Theta_i - \Gamma_i)^T \right. \\ & \times \tilde{\psi}_i(v_i) \theta_i^T \Big) - \frac{2}{c_{i2}} \operatorname{tr} \left((\hat{\Gamma}_i - \Theta_i)^T \right. \\ & \times \left(c_{i1} (\hat{\Gamma}_i - \Theta_i) - c_{i2} \tilde{\psi}_i(v_i) \theta_i^T \right) \Big) \end{aligned} \quad (39)$$

where $\tilde{\psi}_i(v_i) = \psi_i(v_i)/(1 + \psi_i^T(v_i)\psi_i(v_i))$.

Substituting the relation $\Gamma_i = \hat{\Gamma}_i - \tilde{\Gamma}_i$ into (39) yields

$$\begin{aligned} \dot{L}_{iv} = & \tilde{v}_i^T \left(\tilde{\Gamma}_i^T \psi_i(v_i) - K_{i1} \tilde{v}_i - \epsilon_i \right) - 2 \operatorname{tr} \left(\tilde{\Gamma}_i^T \tilde{\psi}_i(v_i) \theta_i^T \right) \\ & - \frac{2c_{i1}}{c_{i2}} \operatorname{tr} \left((\hat{\Gamma}_i - \Theta_i)^T (\hat{\Gamma}_i - \Theta_i) \right) \\ & + 4 \operatorname{tr} \left((\hat{\Gamma}_i - \Theta_i)^T \tilde{\psi}_i(v_i) \theta_i^T \right). \end{aligned} \quad (40)$$

Substituting the relationship $\tilde{\Gamma}_i^T \psi_i(v_i) = \theta_i + \epsilon_i$ derived from (9) into (40) yields

$$\begin{aligned} \dot{L}_{iv} = & \tilde{v}_i^T (\theta_i - K_{i1} \tilde{v}_i) - 2\bar{\psi}_i(v_i) \operatorname{tr} (\theta_i \theta_i^T + \epsilon_i \theta_i^T) \\ & - \frac{2c_{i1}}{c_{i2}} \operatorname{tr} \left((\hat{\Gamma}_i - \Theta_i)^T (\hat{\Gamma}_i - \Theta_i) \right) \\ & + 4\bar{\psi}_i(v_i) \operatorname{tr} \left((\hat{\Gamma}_i - \Theta_i)^T \psi_i(v_i) \theta_i^T \right). \end{aligned} \quad (41)$$

Furthermore, applying the square completion technique under the condition $c_{i1} \geq 2c_{i2}$ yields

$$\begin{aligned} \dot{L}_{iv} \leq & \|\tilde{v}_i\| \|\theta_i\| - \lambda_{\min}(K_{i1}) \|\tilde{v}_i\|^2 - \bar{\psi}_i(v_i) \|\theta_i\|^2 \\ & - \frac{2c_{i1}}{c_{i2}} \|\hat{\Gamma}_i - \Theta_i\|_F^2 \\ & - \bar{\psi}_i(v_i) \left(2\|\hat{\Gamma}_i - \Theta_i\|_F \psi_i(v_i) - \|\theta_i\| \right)^2. \end{aligned} \quad (42)$$

Based on Young's inequality, we have

$$\|\tilde{v}_i\| \|\theta_i\| \leq \frac{\lambda_{\min}(K_{i1})}{2} \|\tilde{v}_i\|^2 + \frac{1}{2\lambda_{\min}(K_{i1})} \|\theta_i\|^2. \quad (43)$$

Substituting (43) into (42) yields

$$\begin{aligned} \dot{L}_{iv} \leq & -\frac{\lambda_{\min}(K_{i1})}{2} \|\tilde{v}_i\|^2 + \left(\frac{1}{2\lambda_{\min}(K_{i1})} - \bar{\psi}_i(v_i) \right) \|\theta_i\|^2 \\ & - \frac{2c_{i1}}{c_{i2}} \bar{\psi}_i(v_i) \|\hat{\Gamma}_i - \Theta_i\|_F^2. \end{aligned} \quad (44)$$

Provided that the condition $1/(2\lambda_{\min}(K_{i1})) - \bar{\psi}_i(v_i) \leq 0$ holds, it follows from (44) that $\dot{L}_{iv} \leq 0$. This implies that $L_{iv}(t) \leq L_{iv}(0)$, which guarantees $\|\tilde{v}_i\| \leq \sqrt{2L_{iv}(0)}$ and $\|\Theta_i - \Gamma_i\|_F, \|\hat{\Gamma}_i - \Theta_i\|_F \leq \sqrt{c_{i2}L_{iv}(0)}$. By applying the triangle inequality to the decomposition $\tilde{\Gamma}_i = (\Theta_i - \Gamma_i) + (\hat{\Gamma}_i - \Theta_i)$, we obtain $\|\tilde{\Gamma}_i\|_F \leq 2\sqrt{c_{i2}L_{iv}(0)}$. According to Definition 1, these results confirm that the errors $\tilde{v}_i$ and $\tilde{\Gamma}_i$ are uniformly bounded, which completes the proof.

Theorem 2: Consider the distributed reference trajectory observer described by (15) and the adaptive weight update law (16). Under Assumption 1, the distributed observer error $\tilde{r}_i$ and the weight estimation error $\tilde{S}_i = \hat{S}_i - S$ are UUB, provided that $\varrho_{i1} > 1$ and $c_{i3} > c_{i4} \|\psi_0(\hat{r}_i)\|^2$.

Proof: Consider the following Lyapunov candidate function:

$$L_r = \frac{1}{2} \delta^T (\mathcal{H}(\eta) \otimes I_n) \delta + \frac{1}{2c_{i4}} \sum_{i=1}^N \text{tr} \left(\tilde{S}_i^T \tilde{S}_i \right). \quad (45)$$

Taking the time derivative of L_r and substituting the adaptive update law (16) yields

$$\begin{aligned} \dot{L}_r = & \sum_{i=1}^N \tilde{r}_i^T \left(\tilde{S}_i^T \psi_0(\hat{r}_i) - K_{i2} \tilde{r}_i + \Omega_i \right) \\ & - \frac{1}{c_{i4}} \sum_{i=1}^N \text{tr} \left(c_{i3} \tilde{S}_i^T \hat{S}_i + c_{i4} \tilde{S}_i^T \tilde{\psi}_0(\hat{r}_i) \tilde{r}_i^T \right). \end{aligned} \quad (46)$$

Using the property $\text{tr}(\tilde{S}_i^T \psi_0(\hat{r}_i) \tilde{r}_i^T) = \tilde{r}_i^T \tilde{S}_i^T \psi_0(\hat{r}_i)$, and (46) can be rewritten as

$$\begin{aligned} \dot{L}_r = & \sum_{i=1}^N \tilde{r}_i^T (\Omega_i - K_{i2} \tilde{r}_i) - \frac{c_{i3}}{c_{i4}} \sum_{i=1}^N \text{tr} \left(\tilde{S}_i^T \hat{S}_i \right) \\ & + \sum_{i=1}^N \tilde{r}_i^T \tilde{S}_i^T \psi_0(\hat{r}_i) \psi_0^T(\hat{r}_i) \tilde{\psi}_0(\hat{r}_i). \end{aligned} \quad (47)$$

By applying Young's Inequality, we obtain

$$\tilde{r}_i^T \Omega_i \leq \frac{1}{2} \|\tilde{r}_i\|^2 + \frac{1}{2} \bar{\Omega}_i^2 \quad (48)$$

$$\tilde{r}_i^T \tilde{S}_i^T \tilde{\psi}_0(\hat{r}_i) \leq \frac{1}{2} \|\tilde{r}_i\|^2 + \frac{1}{2} \|\psi_0(\hat{r}_i)\|^2 \|\tilde{S}_i\|_F^2 \quad (49)$$

$$- \frac{c_{i3}}{c_{i4}} \text{tr} \left(\tilde{S}_i^T \hat{S}_i \right) \leq - \frac{c_{i3}}{2c_{i4}} \|\tilde{S}_i\|_F^2 + \frac{c_{i3}}{2c_{i4}} \|S\|_F^2. \quad (50)$$

Note that $0 \leq \psi_0^T(\hat{r}_i) \tilde{\psi}_0(\hat{r}_i) < 1$. Substituting (48)–(50) into (47) yields

$$\begin{aligned} \dot{L}_r \leq & - \sum_{i=1}^N (K_{i2} - 1) \|\tilde{r}_i\|^2 \\ & - \sum_{i=1}^N \left(\frac{c_{i3}}{2c_{i4}} - \frac{1}{2} \|\psi_0(\hat{r}_i)\|^2 \right) \|\tilde{S}_i\|_F^2 + \Omega_r \end{aligned} \quad (51)$$

where $\Omega_r = \sum_{i=1}^N \left(\frac{1}{2} \bar{\Omega}_i^2 + \frac{c_{i3}}{2c_{i4}} \|S\|_F^2 \right)$.

Let $G_1 = \min_{i=1,2,\dots,N} (K_{i2} - 1)$ and $G_2 = \min_{i=1,2,\dots,N} \left(\frac{c_{i3}}{2c_{i4}} - \frac{1}{2} \|\psi_0(\hat{r}_i)\|^2 \right)$. Provided that the parameters are selected such that $\varrho_{i1} > 1$ and $c_{i3} > c_{i4} \|\psi_0(\hat{r}_i)\|^2$, we have $G_1 > 0$ and $G_2 > 0$. Consequently, $\dot{L}_r < 0$ whenever either $\|\tilde{r}\| > \sqrt{\Omega_r/G_1}$ or $\|\tilde{S}\|_F > \sqrt{\Omega_r/G_2}$. According to Definition 1, this implies that the errors $\tilde{r}$ and $\tilde{S}$ are UUB.

Under Assumption 1, for every piecewise constant switching signal $\eta \in \mathcal{P}$, the graph contains a spanning tree rooted at the leader, ensuring that $\mathcal{H}(\eta)$ is positive definite. Since the set of admissible topologies $\mathcal{P}$ is finite, there exists $\min_{k \in \mathcal{P}} \lambda_{\min}(\mathcal{H}(k)) > 0$. Recalling the error relationship $\tilde{r}(t) = (\mathcal{H}(\eta) \otimes I_n) \delta(t)$ yields

$$\|\tilde{r}(t)\| \geq \min_{k \in \mathcal{P}} \sigma_{\min}(\mathcal{H}(k)) \|\delta(t)\|. \quad (52)$$

Consequently, the established boundedness of $\|\tilde{r}(t)\|$ directly guarantees the boundedness of the distributed trajectory estimation error $\|\delta(t)\|$.

Theorem 3: Consider the second-order nonlinear MASs (1), along with the FLS-based estimate (7), the reference trajectory observer (15), the adaptive critic and actor network weight update laws (33) and (37), and the distributed formation controller (36). Under Assumptions 1, the closed-loop error dynamics (20) and (22) are guaranteed to be UUB, provided that there exist gain matrices K_{i3}, K_{i4} , positive scalars b_{i1}, b_{i2}, b_{i3} , and a symmetric positive definite matrix Q_i such that the following inequalities are satisfied:

$$\begin{cases} \lambda_{\min}(K_{i3}) - \frac{1}{2} \|K_{i4}\|^2 - \frac{1}{2} \geq 0 \\ \lambda_{\min}(K_{i3}) - \frac{1}{2} \|A_i(x_i)\|^2 - 3 \geq 0 \\ b_{i1} + b_{i2} - b_{i3} \geq 0 \\ b_{i3} - \|\tilde{Q}_i\|^2 \geq 0 \end{cases} \quad (53)$$

where $\tilde{Q}_i = B_i(x_i) Q_i^{-1} B_i^T(x_i) \phi_{e_i}^T(e_i)$.

Proof: Choose the Lyapunov candidate function as

$$L_{ie} = \frac{1}{2} \left(\rho_i^T \rho_i + e_i^T e_i + \text{tr} \left(\tilde{W}_{ic}^T \tilde{W}_{ic} \right) + \text{tr} \left(\tilde{W}_{ia}^T \tilde{W}_{ia} \right) \right) \quad (54)$$

where $\tilde{W}_{ic} = \hat{W}_{ic} - W_i$ and $\tilde{W}_{ia} = \hat{W}_{ia} - W_i$. Taking the time derivative of L_{ie} yields

$$\dot{L}_{ie} = \rho_i^T \dot{\rho}_i + e_i^T \dot{e}_i + \text{tr} \left(\tilde{W}_{ic}^T \dot{\tilde{W}}_{ic} \right) + \text{tr} \left(\tilde{W}_{ia}^T \dot{\tilde{W}}_{ia} \right). \quad (55)$$

It follows from (20) and (21) that

$$\rho_i^T \dot{\rho}_i = \rho_i^T A_i(x_i) e_i - \rho_i^T K_{i3} \rho_i$$

$$\leq \left(\frac{1}{2} - \lambda_{\min}(K_{i3})\right) \|\rho_i\|^2 + \frac{1}{2} \|A_i(x_i)\|^2 \|e_i\|^2. \quad (56)$$

Similarly, for the velocity tracking error e_i , we have

$$\begin{aligned} e_i^T \dot{e}_i &= -\frac{1}{2} e_i^T \tilde{Q}_i \hat{W}_{ia} - e_i^T K_{i3} e_i - e_i^T K_{i4} \rho_i \\ &\quad - e_i^T \tilde{\Gamma}_i^T(t) \psi_i(v_i) - e_i^T \dot{v}_i^d + e_i^T \omega_i + e_i^T \epsilon_i \\ &\leq -(\lambda_{\min}(K_{i3}) - 3) \|e_i\|^2 + \frac{1}{2} \|K_{i4}\|^2 \|\rho_i\|^2 \\ &\quad + \frac{1}{2} \|\tilde{Q}_i\|^2 \|\hat{W}_{ia}\|^2 + \frac{1}{2} \|\tilde{\Gamma}_i^T(t) \psi_i(v_i)\|^2 \\ &\quad + \frac{1}{2} \|\dot{v}_i^d\|^2 + \frac{1}{2} \|\omega_i\|^2 + \frac{1}{2} \|\epsilon_i\|^2. \end{aligned} \quad (57)$$

For the weight estimation errors $\tilde{W}_{ic}$ and $\tilde{W}_{ia}$, according to the adaptive laws (33) and (37), we have

$$\begin{aligned} &\text{tr} \left(\tilde{W}_{ic}^T \dot{\tilde{W}}_{ic} \right) + \text{tr} \left(\tilde{W}_{ia}^T \dot{\tilde{W}}_{ia} \right) \\ &= -\text{tr} \left(b_{i1} \tilde{W}_{ic}^T \phi_i(e_i) \phi_i^T(e_i) \hat{W}_{ic} \right) \\ &\quad - \text{tr} \left((b_{i2} - b_{i3}) \tilde{W}_{ia}^T \phi_i(e_i) \phi_i^T(e_i) \hat{W}_{ic} \right. \\ &\quad \left. + b_{i3} \tilde{W}_{ia}^T \phi_i(e_i) \phi_i^T(e_i) \hat{W}_{ia} \right). \end{aligned} \quad (58)$$

Applying Young's inequality yields

$$\tilde{W}_{ic}^T \phi_i \phi_i^T \hat{W}_{ic} \leq \frac{1}{2} \tilde{W}_{ic}^T \phi_i \phi_i^T \tilde{W}_{ic} + \frac{1}{2} \hat{W}_{ic}^T \phi_i \phi_i^T \hat{W}_{ic} \quad (59)$$

$$\tilde{W}_{ia}^T \phi_i \phi_i^T \hat{W}_{ic} \leq \frac{1}{2} \tilde{W}_{ia}^T \phi_i \phi_i^T \tilde{W}_{ia} + \frac{1}{2} \hat{W}_{ic}^T \phi_i \phi_i^T \hat{W}_{ic}. \quad (60)$$

Using $\tilde{W}_{ia} = \hat{W}_{ia} - W_i$, we can get

$$\begin{aligned} \tilde{W}_{ia}^T \phi_i \phi_i^T \hat{W}_{ia} &= \frac{1}{2} \tilde{W}_{ia}^T \phi_i \phi_i^T \tilde{W}_{ia} + \frac{1}{2} \hat{W}_{ia}^T \phi_i \phi_i^T \hat{W}_{ia} \\ &\quad - \frac{1}{2} W_i^T \phi_i \phi_i^T W_i. \end{aligned} \quad (61)$$

Substituting (56)–(61) into (55) leads to

$$\begin{aligned} \dot{L}_{ie} &\leq -\left(\lambda_{\min}(K_{i3}) - \frac{1}{2} \|K_{i4}\|^2 - \frac{1}{2}\right) \|\rho_i\|^2 \\ &\quad - \left(\lambda_{\min}(K_{i3}) - \frac{1}{2} \|A_i(x_i)\|^2 - 3\right) \|e_i\|^2 \\ &\quad - \frac{b_{i1}}{2} \|\phi_i(e_i)\|^2 \|\tilde{W}_{ic}\|^2 - \frac{b_{i2}}{2} \|\phi_i(e_i)\|^2 \|\tilde{W}_{ia}\|^2 \\ &\quad - \frac{1}{2} (b_{i1} + b_{i2} - b_{i3}) \|\phi_i(e_i)\|^2 \|\hat{W}_{ic}\|^2 \\ &\quad - \frac{1}{2} \left(b_{i3} - \|\tilde{Q}_i\|^2\right) \|\phi_i(e_i)\|^2 \|\hat{W}_{ia}\|^2 \\ &\quad + \frac{1}{2} \|\tilde{\Gamma}_i^T(t) \psi_i(v_i)\|^2 + \frac{1}{2} \|\dot{v}_i^d\|^2 + \frac{1}{2} \bar{\omega}_i^2 \\ &\quad + \frac{1}{2} \bar{\epsilon}_i^2 + \frac{b_{i3}}{2} \|\phi_i(e_i)\|^2 \|W_i\|^2. \end{aligned} \quad (62)$$

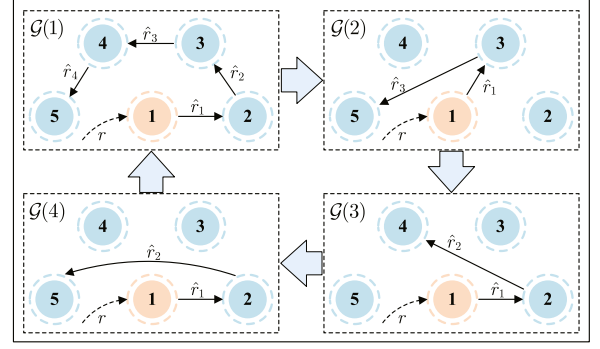


Fig. 2. Switching communication topology of USVs formation.

Under the conditions in (53), (62) can be rewritten as

$$\dot{L}_{ie} \leq -\Phi_i L_{ie} + \Lambda_i \quad (63)$$

where $\Phi_i = \min\{2\lambda_{\min}(K_{i3}) - \|K_{i4}\|^2 - 1, 2\lambda_{\min}(K_{i3}) - \|A_i(x_i)\|^2 - 6, b_{i1}\|\phi_i(e_i)\|^2, b_{i2}\|\phi_i(e_i)\|^2\}$, $\Lambda_i = (\|\dot{v}_i^d\|^2 + \bar{\omega}_i^2 + \bar{\epsilon}_i^2 + \|\tilde{\Gamma}_i^T(t) \psi_i(v_i)\|^2 + b_{i3}\|\phi_i(e_i)\|^2 \|W_i\|^2)/2$.

Furthermore, it follows from (63) that:

$$L_{ie}(t) \leq L_{ie}(0)e^{-\Phi_i t} + \frac{\Lambda_i}{\Phi_i} (1 - e^{-\Phi_i t}) \quad (64)$$

which implies that $\|\rho_i\|, \|e_i\|, \|\tilde{W}_{ic}\|, \|\tilde{W}_{ia}\| \leq \sqrt{2\Lambda_i/\Phi_i}$.

According to Definition 1, these bounds indicate that the closed-loop error dynamics (20) and (22) are UUB, which completes the proof.

IV. SIMULATION EXAMPLE

In this section, the effectiveness and robustness of the proposed control strategy are assessed through Monte Carlo simulations on a formation of $N = 5$ USVs, where $M = 100$ independent trials are performed over a simulation time of $T = 100$ s to evaluate system performance under statistical uncertainties. For each USV, the state vectors are defined as $x_i = [x_{i1}, x_{i2}, x_{i3}]^T$ and $v_i = [v_{i1}, v_{i2}, v_{i3}]^T$, where the detailed dynamic model parameters are adopted from [6]. To assess robustness against model mismatches, random parametric uncertainties are introduced by perturbing the inertial and damping matrices as $\tilde{M}_i = (0.9 + 0.2 \text{rand}(1))M_i$ and $\tilde{D}_i = (0.9 + 0.2 \text{rand}(1))D_i$, where M_i and D_i denote the nominal inertial and damping matrices. The reference trajectory $r(t)$ is generated by $r(t) = [0.14\sqrt{3}t - 1.5 \sin(0.07t), 0.14t + 1.5\sqrt{3} \sin(0.07t), \text{atan2}(0.14 + 0.105\sqrt{3} \cos(0.07t), 0.14\sqrt{3} - 0.105 \cos(0.07t))]$. A switching communication topology $\mathcal{G}(\eta)$, where $\eta \in \{1, 2, 3, 4\}$ denotes the active communication graph shown in Fig. 2, is employed and periodically alternates among four subgraphs with a switching period of 1.0 s.

The external environmental disturbance $\omega_i(t)$ is described by $\omega_i(t) = \gamma_i([0.4 \sin(0.5t), 0.2 \cos(0.4t), 0.05 \sin(0.1t)]^T + 0.05 \text{rand}(3))$, where $\gamma_i = 0.1$ is the disturbance intensity. The FLS is constructed with $m = 6$ fuzzy rules, where the centers are evenly spaced in the interval $[-1, 1]$. The number of neurons in the actor and critic networks is set to $q = 8$. The

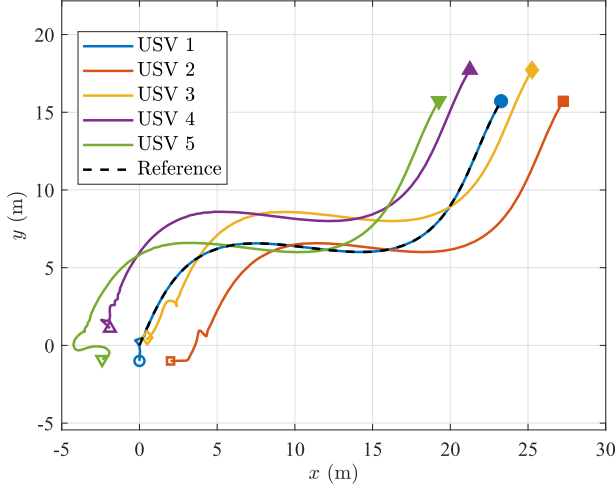


Fig. 3. Actual trajectories of the five USVs.

TABLE I
PERFORMANCE METRICS UNDER DIFFERENT DISTURBANCE INTENSITIES

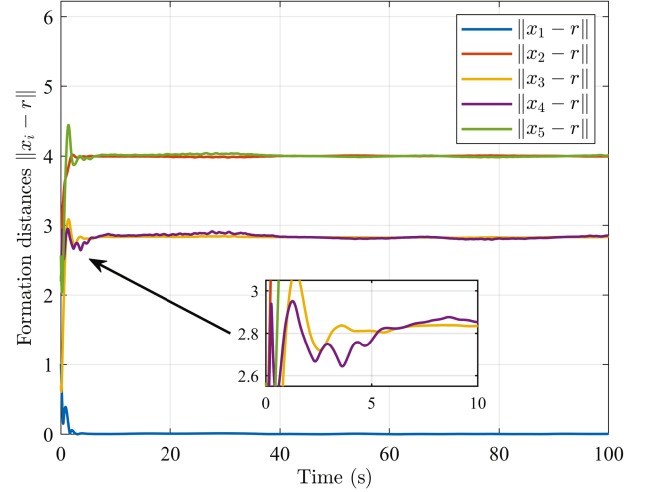
Disturbance γ_i	1.0	2.0	3.0	4.0	5.0
Observer MSE ₁	0.0071	0.0076	0.0075	0.0072	0.0069
Approximation MSE ₂	1.8930	1.8793	1.9455	1.9375	1.8460
Tracking MSE ₃	0.0211	0.0220	0.0248	0.0281	0.0325

TABLE II
PERFORMANCE METRICS UNDER DIFFERENT COMMUNICATION DELAYS

Delay τ_i	0.1	0.2	0.3	0.4	0.5
Observer MSE ₁	0.0160	0.0329	0.0546	0.0811	0.1162
Approximation MSE ₂	4.2202	6.1261	7.6183	8.4901	9.0602
Tracking MSE ₃	0.0289	0.0471	0.0692	0.0963	0.1336

initial weights for the FLS, actor, and critic networks are initialized to 0 or small values. The gain matrices are selected as $K_{i1} = 1.1I_3$, $K_{i3} = 3.6I_3$, and $K_{i4} = 2.4I_3$. The parameters determining K_{i2} are selected as $\varrho_{i1} = 2.1$, $\varrho_{i2} = 23.9$, and $\varrho_{i3} = 0.8$. The weighting matrix chosen as $Q_i = 1.9I_3$. The learning rates are set as $b_{i1} = 2.0$, $b_{i2} = 2.0$, $b_{i3} = 2.0$, $c_{i1} = 44.0$, and $c_{i2} = 21.0$. The desired formation displacements for the five USVs are defined as $d_1 = [0, 0, 0]^T$, $d_2 = [4, 0, 0]^T$, $d_3 = [2, 2, 0]^T$, $d_4 = [-2, 2, 0]^T$, $d_5 = [-4, 0, 0]^T$. The initial positions of the USVs are set as $x_1(0) = [0, -1, 0]^T$, $x_2(0) = [2, -1, 0]^T$, $x_3(0) = [0.5, 0.5, 0]^T$, $x_4(0) = [-1.9, 1.1, 0]^T$, $x_5(0) = [-2.4, -0.9, 0]^T$. The initial velocities are set to $v_i(0) = [0, 0, 0]^T$ for all USVs.

The simulation results are presented in Figs. 3–10 and Tables I and II. Fig. 3 visualizes the actual trajectories of the five USVs together with the reference trajectory. Despite starting from widely dispersed initial positions, all agents rapidly align their motion and converge toward the reference trajectory under the proposed distributed control strategy. The USVs not only track the dynamic reference smoothly but also form and preserve the prescribed geometric formation pattern dictated by the displacement vectors d_i . This result qualitatively demonstrates the effectiveness of the proposed formation controller in handling

Fig. 4. Time evolution of the formation distances $\|x_i - r\|$ for USVs.

second-order nonlinear dynamics and achieving coordinated motion.

Fig. 4 illustrates the temporal evolution of the relative distances between each USV and the reference trajectory. It can be observed that the formation distances for all USVs converge rapidly to their respective steady-state values within approximately 5 s, demonstrating the fast transient response achieved by the proposed control scheme.

To evaluate the overall performance of the proposed method, three MSE-based metrics are introduced to assess observer accuracy, dynamics approximation quality, and formation tracking error. The observer MSE₁ is defined as

$$\text{MSE}_1 = \frac{1}{MNT} \sum_{k=1}^M \sum_{i=1}^N \int_0^T \|\hat{r}_i^{(k)}(t) - r(t)\|^2 dt \quad (65)$$

where $\hat{r}_i^{(k)}(t)$ denotes the estimated reference trajectory in the k th run. The approximation MSE₂ is defined as

$$\text{MSE}_2 = \frac{1}{MNT} \sum_{k=1}^M \sum_{i=1}^N \int_0^T \|\hat{f}_i^{(k)}(v_i) - f_i^{(k)}(v_i)\|^2 dt \quad (66)$$

where $\hat{f}_i^{(k)}(v_i)$ is the approximated dynamics and $f_i^{(k)}(v_i)$ represents the actual dynamics in the k th run. The formation tracking MSE₃ is defined as

$$\text{MSE}_3 = \frac{1}{MNT} \sum_{k=1}^M \sum_{i=1}^N \int_0^T \|x_i^{(k)}(t) - r(t) - d_i\|^2 dt \quad (67)$$

where $x_i^{(k)}(t)$ represents the actual position in the k th run.

Table I summarizes the performance metrics of the system under varying disturbance intensities. As the disturbance level γ_i increases, the observer error MSE₁ and approximation error MSE₂ remain virtually invariant, and the tracking error MSE₃ shows only a marginal increase. These results confirm the exceptional robustness of the proposed control scheme, demonstrating the capacity to effectively suppress external disturbances and maintain high-precision formation tracking.

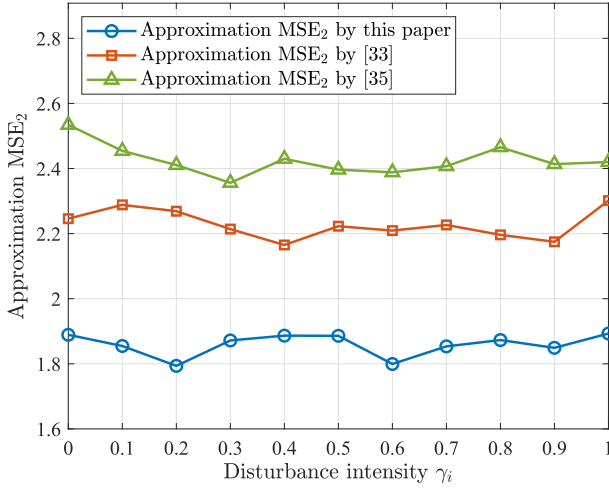


Fig. 5. Approximation MSE_2 under different disturbance intensities γ_i .

Table II evaluates the performance metrics of the proposed control scheme under varying communication delays. For USVs operating under typical communication conditions like standard VHF radio networks [5] and maritime satellite links [28], delays between $\tau_i = 0.1$ s and 0.5 s reflect communication constraints ranging from mild to severe. Although the tracking error MSE_3 increases to 0.1336 as the delay τ_i grows, such a position deviation is sufficiently small compared to the prescribed formation displacements d_i , ensuring that the desired geometric configuration is maintained. These results substantiate the practical robustness of the proposed architecture, confirming that it can achieve effective formation tracking even under maritime communication constraints.

Fig. 5 presents a comparative analysis of the approximation MSE_2 against varying disturbance intensity coefficients $\gamma_i \in [0, 1.0]$. The proposed predictor-based fuzzy learning approach consistently achieves the lowest approximation error, whereas the methods reported in [33] and [35] exhibit substantially higher error levels, with the average error reduced by more than 16%. The observed performance superiority demonstrates that the proposed two-layer learning mechanism substantially improves the ability of the FLS to identify unknown nonlinear dynamics and suppress external perturbations.

Fig. 6 illustrates the convergence time of the approximation error under different disturbance intensities γ_i , where convergence is defined as the time required for the error to decrease below 0.1 . The results show that the proposed predictor-based method exhibits strong robustness, consistently achieving a rapid convergence of approximately 3.6 s across all tested disturbance levels. In contrast, the performance of the comparison methods deteriorates markedly as the disturbance increases: the method in [35] is highly sensitive to perturbations and fails to meet the convergence criterion once $\gamma_i \geq 0.4$, and the method in [33] also experiences a substantial increase in convergence time when $\gamma_i \geq 0.6$. This result provides strong evidence that the proposed two-layer learning mechanism effectively enables rapid and high precision approximation of unknown nonlinear dynamics in complex environments.

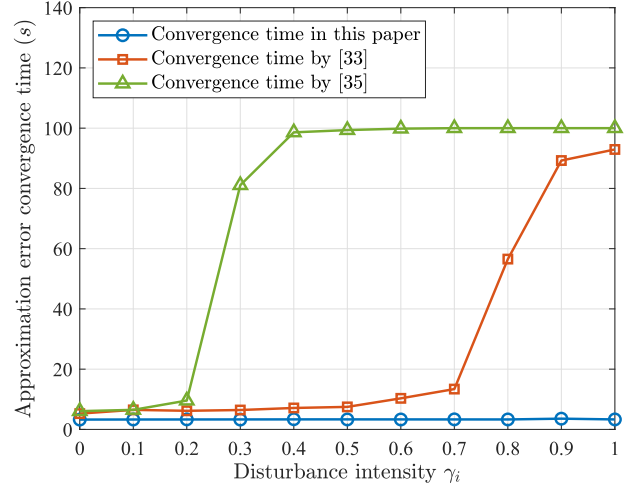


Fig. 6. Comparison of approximation error convergence time under different disturbance intensities γ_i .

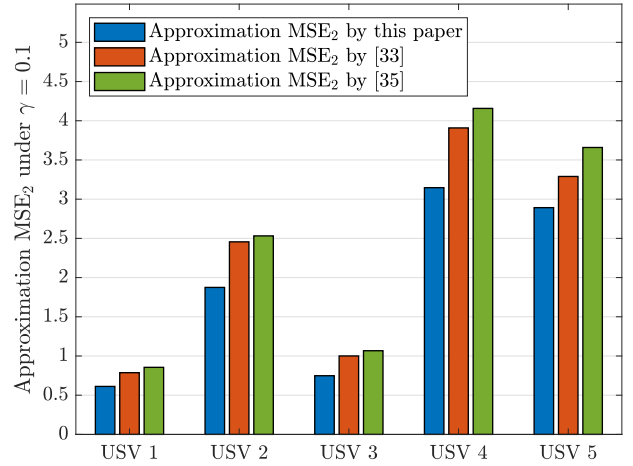


Fig. 7. Approximation MSE_2 for USVs under disturbance intensity $\gamma_i = 0.1$.

Fig. 7 presents the approximation MSE_2 for each individual USV under disturbance intensity $\gamma_i = 0.1$. The results show that the proposed predictor-based fuzzy learning strategy consistently achieves the lowest approximation error across all five USVs, reducing the MSE_2 by at least 12% relative to the methods in [33] and [35]. This uniform improvement across all agents highlights the effectiveness of the two-layer learning mechanism in enhancing the approximation accuracy of unknown nonlinear dynamics for every USV.

Fig. 8 illustrates the variation of the tracking MSE_3 with respect to the disturbance intensity $\gamma_i \in [0, 1.0]$. Across all disturbance levels, the proposed predictor-based fuzzy learning strategy maintains an approximately constant error of 0.02 , whereas the methods in [36] and [37] produce markedly higher errors, resulting in an average tracking error reduction of more than 54%. Fig. 9 presents a comparative analysis of the tracking error convergence time under disturbance intensities varying from $\gamma_i = 0$ to 1.0 , where convergence is defined as the time required for the tracking error to settle within 0.1 . The proposed

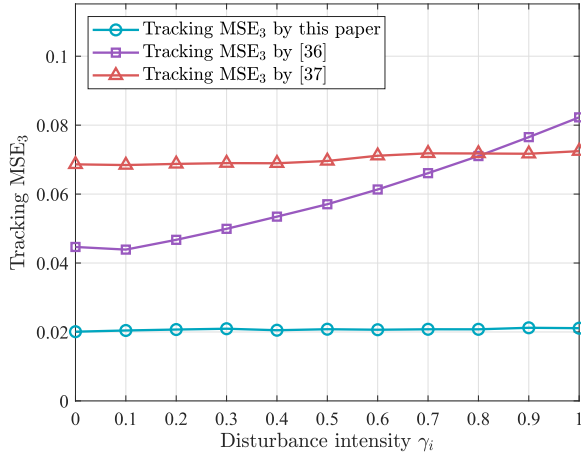


Fig. 8. Tracking MSE₃ under different disturbance intensities γ_i .

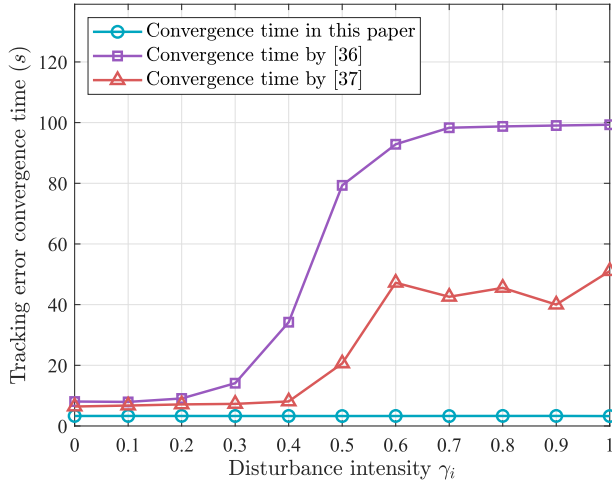


Fig. 9. Comparison of tracking error convergence time under different disturbance intensities γ_i .

method demonstrates strong robustness and efficiency, sustaining a consistently low convergence time of approximately 3.4 s across the full disturbance range. In contrast, the method in [36] fails to achieve convergence within the simulation horizon when $\gamma_i \geq 0.7$, and the approach in [37] becomes increasingly unstable, with convergence times fluctuating between 40 s and 50 s for $\gamma_i > 0.4$. From Figs. 8 and 9, the pronounced performance advantage indicates that the actor-critic learning control framework, augmented with the compensatory term, effectively alleviates the approximation burden of the critic network and ensures reliable tracking accuracy even under increasingly severe perturbations.

Fig. 10 illustrates the tracking MSE₃ for each individual USV under a disturbance intensity of $\gamma_i = 0.1$. The results indicate that the proposed formation control strategy consistently yields the lowest tracking errors across all five USVs, reducing the MSE₃ by at least 37% relative to the methods in [36] and [37]. This consistent performance advantage among all USVs confirms that incorporating the compensatory term into the actor-critic framework effectively optimizes the control policy for each

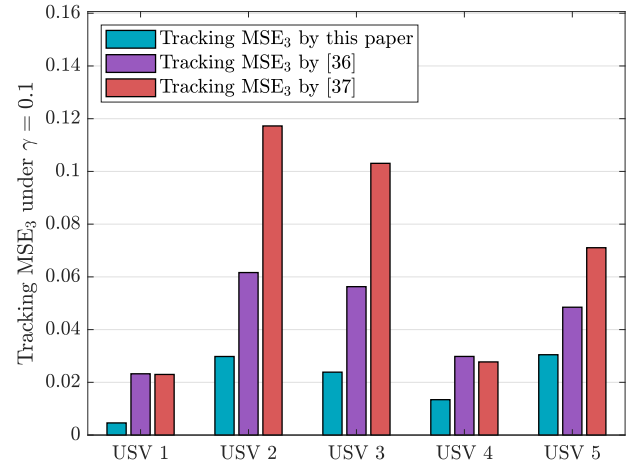


Fig. 10. Tracking MSE₃ for USVs under disturbance intensity $\gamma_i = 0.1$.

member of the formation, thereby ensuring precise formation maintenance.

V. CONCLUSION

This article has presented a novel distributed formation control architecture for second-order nonlinear MASs subject to unknown dynamics and limited communication. The core contribution lies in the seamless integration of a predictor-based fuzzy learning strategy with an actor-critic reinforcement learning framework. By leveraging velocity prediction errors, the proposed two-layer learning mechanism significantly accelerates the approximation of unknown nonlinearities. Furthermore, the introduction of a structural compensatory term reformulates the value function approximation into a residual learning task, thereby expediting policy convergence. Extensive simulations on USV formations confirm that the proposed method achieves superior tracking accuracy and rapid convergence even under high intensity disturbances.

REFERENCES

- [1] D. Jin and Z. Xiang, "Predefined-time consensus for second-order nonlinear multiagent systems via sliding mode technique," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 8, pp. 4534–4541, Aug. 2024.
- [2] R. Zhang, S. Li, C. K. Ahn, and M. Chadli, "Fully distributed adaptive fuzzy consensus for heterogeneous switched nonlinear multiagent systems under state-dependent switchings," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 2, pp. 607–620, Feb. 2024.
- [3] J. Zhang, Y. Fu, and J. Fu, "Optimal formation control of second-order heterogeneous multiagent systems using adaptive predefined-time strategy," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 4, pp. 2390–2402, Apr. 2024.
- [4] P. Gong and C. Ki Ahn, "Robust adaptive fuzzy fixed-time distributed time-varying optimization in unbalanced nonlinear second-order multiagent networks," *IEEE Trans. Fuzzy Syst.*, vol. 33, no. 8, pp. 2730–2742, Aug. 2025.
- [5] D. Palma, "Enabling the maritime Internet of Things: Coap and 6lowpan performance over vhf links," *IEEE Internet Things J.*, vol. 5, no. 6, pp. 5205–5212, Dec. 2018.
- [6] L. Zhu, J. Miao, J. Liu, E. Tian, and C. Peng, "Neural predictor-based formation control for unmanned surface vehicles under aperiodic dos attacks: A noncooperative game approach," *IEEE Trans. Veh. Technol.*, vol. 74, no. 10, pp. 15013–15025, Oct. 2025.
- [7] J. Liu, S. Mao, L. Zhu, E. Tian, and C. Peng, "Privacy-preserving and collision-free formation control for heterogeneous multiagent systems," *IEEE Trans. Consum. Electron.*, to be published, doi: [10.1109/TCE.2026.3667772](https://doi.org/10.1109/TCE.2026.3667772).

- [8] X. Jiang, L. You, N. Zhang, M. Chi, and H. Yan, "Impulsive formation tracking of nonlinear fuzzy multiagent systems with input saturation constraints," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 10, pp. 5728–5736, Oct. 2024.
- [9] J. Liu, Z. Liu, Y. Li, E. Tian, and C. Peng, "Privacy-protected formation control for unmanned surface vehicles: A neural predictor-based noncooperative game framework," *IEEE Trans. Veh. Technol.*, to be published, doi: [10.1109/TVT.2026.3651837](https://doi.org/10.1109/TVT.2026.3651837).
- [10] S. Sui, D. Shen, S. Tong, and C. L. P. Chen, "State-observer-based adaptive fuzzy event-triggered formation control for nonlinear multiagent system," *IEEE Trans. Emerg. Topics Comput. Intell.*, vol. 8, no. 5, pp. 3327–3338, Oct. 2024.
- [11] W. Wu and S. Tong, "Collision-free adaptive fuzzy formation control for stochastic nonlinear multiagent systems," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 53, no. 9, pp. 5454–5465, Sep. 2023.
- [12] M. Xu, X. Chen, and F. Hao, "Scalable formation control for second-order multiagent systems: An event-triggered predefined-time strategy," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 55, no. 2, pp. 1466–1477, Feb. 2025.
- [13] K. Luo, B. Zhang, and W. Meng, "Formation control of second-order multiagent system via stochastic control input," *IEEE Trans. Ind. Informat.*, vol. 20, no. 4, pp. 5550–5561, Apr. 2024.
- [14] C. Gu, X. Wu, W.-A. Zhang, H. Ni, and S. X. Ding, "Active disturbance rejection formation control for multiagent systems with input constraints," *IEEE Trans. Control Netw. Syst.*, vol. 12, no. 1, pp. 38–50, Mar. 2025.
- [15] J. Zhang, Y. Fu, and J. Fu, "Adaptive finite-time optimal formation control for second-order nonlinear multiagent systems," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 53, no. 10, pp. 6132–6144, Oct. 2023.
- [16] J. Lan, Y.-J. Liu, D. Yu, G. Wen, S. Tong, and L. Liu, "Time-varying optimal formation control for second-order multiagent systems based on neural network observer and reinforcement learning," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 35, no. 3, pp. 3144–3155, Mar. 2024.
- [17] Y. Huang, Z. Meng, and J. Sun, "Distributed constrained optimization for second-order multiagent systems via event-based communication," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 54, no. 9, pp. 5317–5326, Sep. 2024.
- [18] W. Bi, C. Zhang, S. Sui, S. Tong, and C. L. P. Chen, "Fixed-time fuzzy adaptive bipartite output consensus tracking for nonlinear competition mass," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 5, pp. 2775–2785, May 2024.
- [19] Y. Yan, H. Zhang, J. Sun, and Y. Wang, "Sliding mode control based on reinforcement learning for T-S fuzzy fractional-order multiagent system with time-varying delays," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 35, no. 8, pp. 10368–10379, Aug. 2024.
- [20] L. Zha, T. Huang, J. Liu, E. Tian, X. Xie, and C. Peng, "Secure state estimation for interval type-2 fuzzy systems with FDI attacks and event-triggered WTOD protocol," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 55, no. 8, pp. 5320–5331, Aug. 2025.
- [21] J. Wang, J. Liu, Y. Li, C. L. P. Chen, Z. Liu, and F. Li, "Prescribed time fuzzy adaptive consensus control for multiagent systems with dead-zone input and sensor faults," *IEEE Trans. Autom. Sci. Eng.*, vol. 21, no. 3, pp. 4016–4027, Jul. 2024.
- [22] Y. Xu, H. Liang, T. Li, Y. Long, Y. Cheng, and D. Wang, "Adaptive fuzzy resilient control of nonlinear multiagent systems under DoS attacks: A dynamic event-triggered method," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 6, pp. 3568–3580, Jun. 2024.
- [23] H. Shen, W. Zhao, J. Cao, J. H. Park, and J. Wang, "Predefined-time event-triggered tracking control for nonlinear servo systems: A fuzzy weight-based reinforcement learning scheme," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 8, pp. 4557–4569, Aug. 2024.
- [24] J. Wang, J. Wu, H. Shen, J. Cao, and L. Rutkowski, "Fuzzy H_∞ control of discrete-time nonlinear Markov jump systems via a novel hybrid reinforcement Q-learning method," *IEEE Trans. Cybern.*, vol. 53, no. 11, pp. 7380–7391, Nov. 2023.
- [25] H.-Y. Zhu, Y.-X. Li, and S. Tong, "Fuzzy adaptive tracking of constrained nonlinear systems with event-sampling reinforcement learning," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 2, pp. 536–546, Feb. 2024.
- [26] Q. Fu, Z. Pu, Y. Pan, T. Qiu, and J. Yi, "Fuzzy feedback multiagent reinforcement learning for adversarial dynamic multiteam competitions," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 5, pp. 2811–2824, May 2024.
- [27] J. Liu, N. Zhang, Y. Li, X. Xie, E. Tian, and J. Cao, "Learning-based event-triggered tracking control for nonlinear networked control systems with unmatched disturbance," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 53, no. 5, pp. 3230–3240, May 2023.
- [28] D. Agnew, A. Rice-Bladykas, and J. McNair, "Detection of zero-day attacks in a software-defined leo constellation network using enhanced network metric predictions," *IEEE Open J. Commun. Soc.*, vol. 5, pp. 6611–6634, 2024, doi: [10.1109/OJCOMS.2024.3481965](https://doi.org/10.1109/OJCOMS.2024.3481965).
- [29] Y. Meng, C. Liu, Q. Wang, and L. Tan, "Cooperative advantage actor-critic reinforcement learning for multiagent pursuit-evasion games on communication graphs," *IEEE Trans. Artif. Intell.*, vol. 5, no. 12, pp. 6509–6523, Dec. 2024.
- [30] Q. Liu, H. Yan, M. Wang, Z. Li, and S. Liu, "Data-driven optimal bipartite consensus control for second-order multiagent systems via policy gradient reinforcement learning," *IEEE Trans. Cybern.*, vol. 54, no. 6, pp. 3468–3478, Jun. 2024.
- [31] L. Chen, S.-L. Dai, and C. Dong, "Adaptive optimal tracking control of an underactuated surface vessel using actor-critic reinforcement learning," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 35, no. 6, pp. 7520–7533, Jun. 2024.
- [32] J. Lan, Y.-J. Liu, T. Xu, S. Tong, and L. Liu, "Adaptive fuzzy fast finite-time formation control for second-order MASs based on capability boundaries of agents," *IEEE Trans. Fuzzy Syst.*, vol. 30, no. 9, pp. 3905–3917, Sep. 2022.
- [33] K. Li, K. Feng, and Y. Li, "Fuzzy adaptive fault-tolerant formation control for USVs with intermittent actuator faults," *IEEE Trans. Intell. Veh.*, vol. 9, no. 3, pp. 4445–4455, Mar. 2024.
- [34] G. Wen, W. Hao, W. Feng, and K. Gao, "Optimized backstepping tracking surface vessels flexible tracking learning for quadrotor unmanned aerial vehicle system," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 52, no. 8, pp. 5004–5015, Aug. 2022.
- [35] R. Lu et al., "Fuzzy actor-critic reinforcement learning for unmanned surface vessels flexible tracking formation control," *IEEE Trans. Fuzzy Syst.*, vol. 33, no. 10, pp. 3666–3680, Oct. 2025.
- [36] N. Wang, Y. Gao, and X. Zhang, "Data-driven performance-prescribed reinforcement learning control of an unmanned surface vehicle," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 12, pp. 5456–5467, Dec. 2021.
- [37] N. Wang, Y. Gao, H. Zhao, and C. K. Ahn, "Reinforcement learning-based optimal tracking control of an unknown unmanned surface vehicle," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 7, pp. 3034–3045, Jul. 2021.



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